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# Enhanced thermodynamic efficiency in time asymmetric ratchets

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**Abstract.** The energetic efficiency of an overdamped Brownian particle in a sawtooth potential in the presence of time asymmetric forcing is studied in the adiabatic limit. An error made in earlier work on the same problem in the literature is corrected. We find that asymmetry in the potential together with temporal asymmetry in the forcing leads to much enhanced efficiency without fine-tuning of the parameters. The origin of this is traced to the suppression of the backward current. We also present a comparative study of the roles of continuous and discontinuous ratchet forces as regards these measurable quantities. We find that the thermal fluctuations can optimize the energy transduction, the range of parameters, however, being very small. This ratchet model also displays current reversals on tuning of parameters even in the adiabatic regime. The possible relationships between the nature of the currents, entropy production and input energy are also addressed.

**Keywords:** Brownian motion, stochastic processes (theory), molecular motors (theory)

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## Contents



## 1. Introduction

The study of the nature of directed motion induced by random noise in periodic systems in the absence of a bias has attracted wide interest. The rectification of thermal fluctuations has become a major area of research in nonequilibrium statistical mechanics. The presence of unbiased nonequilibrium perturbations, either stochastic or deterministic, together with a broken spatial or temporal asymmetry, plays a key role in obtaining directed motion without violating the second law of thermodynamics. Such systems or ratchets convert nonequilibrium fluctuations into useful work in the presence of a load. Moreover, in these systems, noise plays a constructive role (i.e., transformation of noise in spatially periodic systems into directed current). A large family of models of Brownian ratchets [1]–[5] have been introduced to obtain insight into the basic mechanism of noise rectification. These include flashing ratchets, rocking ratchets, time asymmetric ratchets and frictional ratchets [2]. Numerous studies have been carried out in efforts to understand the nature of currents, their possible reversals and also the efficiency of energy transduction. The results obtained are utilized to develop proper models that efficiently separate particles of microsize and nanosize and also for the development of machines at nanoscales [4]. Such models are also prototypes for explaining the basic mechanism of operation of molecular motors or protein molecules in our cells that transfer cargo and organelles very efficiently in a very noisy environment. This mechanism also has extensions in game theory, under the general name of Parrondo's paradox [6]. These are basically counter-intuitive games based on translation of the dynamics of Brownian particles in a flashing ratchet to gambling games. Here, two losing games (or strategies), when alternated randomly or periodically, give rise to a winning game. These paradoxes have a profound role in several multidisciplinary areas.

With the emergence of a separate subfield called stochastic energetics [7, 8], it has become possible to establish compatibility between the Langevin or Fokker–Planck formalism, which describes stochastic dynamics, and the laws of thermodynamics. Using this framework one can calculate various physical quantities such as the thermodynamic efficiency of energy transduction [9], energy dissipation (hysteresis loss) and entropy (entropy production) [10], thereby providing a new tool for studying systems far from equilibrium.

The intrinsic irreversibility associated with ratchet operation makes the ratchet less efficient. For example, the attained values of the efficiency in flashing and rocking ratchets were found to be below the subpercentage regime. However, it has been shown that at very

J. Stat. Mech. (2005) P04012

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low temperatures fine-tuning of parameters could lead to a larger efficiency, the regime of parameters being very narrow [11]. Optimization of the energetic efficiency of the sawtooth ratchet in the presence of spatial symmetry but in the presence of time symmetric rocking has been worked out in detail in [11]. Moreover, protocols for optimizing the efficiency are given in [11, 12].

Recently Makhnovskii *et al* [13] constructed a special type of flashing ratchet with two asymmetric double-well periodic potential states displaced by half a period. Such flashing ratchet models were found to be highly efficient with efficiency an order of magnitude higher than in earlier models [7]–[9], [14]. The basic idea behind this enhanced efficiency is that even for diffusive Brownian motion the choice of an appropriate potential profile ensures suppression of backward motion and hence reduction in the accompanying dissipation. We had earlier [15] studied the motion of a particle in a rocking ratchet, similar to the case of flashing ratchets [13], by applying a temporally asymmetric but unbiased periodic forcing [16]–[19] in the presence of a sinusoidal potential. The efficiency obtained was very high, far above the subpercentage level (about 30–40% without fine-tuning) in the presence of temporal asymmetry alone.

In [16]–[19] a time asymmetric discontinuous dichotomic forcing of zero average (unbiased) over the period is considered, i.e., the forcing takes two values in a given period and is discontinuous. This is a special case of applied time asymmetric force. It should be noted that if one applies an unbiased biharmonic drive at frequencies $\omega$ and $2\omega$ one can readily generate unidirectional current even in the presence of a periodic symmetric potential. This phenomenon is known as harmonic mixing [20] and has been studied extensively in the context of ratchet dynamics [21], in the problem of kink-assisted directed energy transport in soliton systems [22] etc. Experimentally the harmonic mixing phenomenon has been used in the context of the technologically relevant problem of transport in binary mixtures [23], in the generation of directed photocurrents in semiconductors [24] (for details see section 5.2 of [2]), in the realization of a Brownian motor by using cold atoms in dissipative optical lattices as a cold system [25] etc.

In the present work we study the problem of a particle in a sawtooth potential and make a comparison so as to elucidate the sensitivity of these physical quantities to the smoothness or regularity of the underlying ratchet potential. The important underlying factor is the temporal asymmetry [16]–[19] in the external forcing which leads to noise induced currents in the absence of external bias even for the case of a spatially symmetric potential. In this adiabatically rocked time asymmetric correlation ratchet, a larger force field is applied for a short interval of the time period in one direction as compared to a smaller force for a longer time interval in the other direction; see figure 1. Some qualitative differences between the smooth and piecewise linear ratchet potentials which are observed is discussed. The surprisingly sensitive dependence of the physical quantities such as the unidirectional current on the degree of regularity or smoothness of the ratchets (continuous and discontinuous forces) has been demonstrated by Doering *et al* [26].

Ai *et al* [19] have also studied the same problem of a Brownian particle moving in a periodic sawtooth potential subjected to a temporally asymmetric periodic rocking. However, there is an error in the expression for the energy per unit time that a ratchet gets from the external force or, in other words, the input energy [15]. In this work we take into account this correction and have calculated the efficiency and other physical quantities and presented our results. We find that the temporal asymmetry in driving enhances the

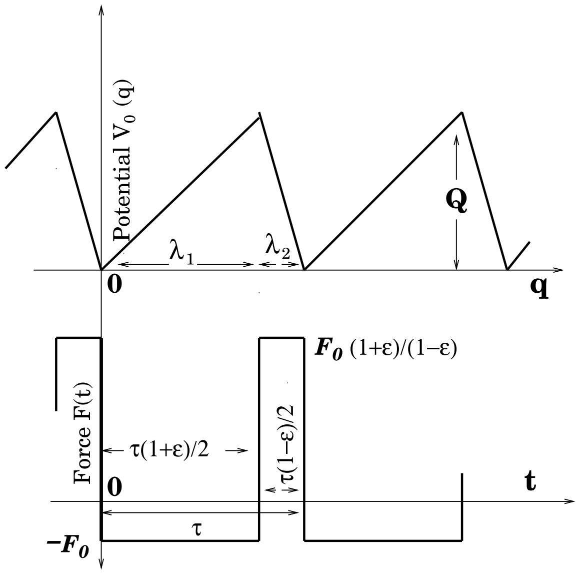


**Figure 1.** Schematic representation of the potential and the external force $F(t)$ as a function of space and time respectively.

efficiency in a very significant manner even for a spatially symmetric potential. Also, in the presence of spatial asymmetry in the potential, the efficiency is found to be almost 90% at low temperature. Current reversals are also observed in the parameter space of operation even in the adiabatic regime.

We also present our analysis of the behaviour of the entropy production, current and input energy with temperature in this ratchet system. In the absence of any bias the noise induced currents show a peak with temperature. The question that naturally arises is whether this peak is related to the underlying resonance (stochastic resonance [27]) due to the synchronization of the position of the particle with the external drive induced by the noise. Our analysis of the input energy $E_{\text{in}}$ rules out the possibility of the presence of any resonance features in the dynamics of the position of the particle in these systems in the adiabatic regime [10, 28]. This follows from earlier works which show that the existence of stochastic resonance in the dynamics of the particle is revealed by a peak in the input energy [29, 30].

The onset of unidirectional currents in ratchet systems can also be viewed as an example of temporal order coming out of disorder. This can happen only at the expense of an overall increase in the entropy production in the system along with its environment. Thus one expects a correlation between the maxima in current production and the maxima in entropy production. However, our results show that the maxima in current and entropy production do not correlate with each other.

## 2. The model

A simple model for our ratchet system is described by the stochastic differential equation (Langevin equation) for a Brownian particle in the overdamped regime. This is given



J. Stat. Mech. (2005) P04012

by [31]

$$\dot{q} = -\left[\frac{V'(q) - F(t) + L}{\gamma}\right] + \xi(t), \tag{1}$$

where $\xi(t)$ is a randomly fluctuating Gaussian thermal noise with zero mean and correlation, $\langle \xi(t)\xi(t')\rangle = (2\,k_B T/\gamma)\delta(t-t')$.

In the present work we consider the piecewise linear ratchet potential, figure 1, as in the case of Magnasco *et al* [32] with periodicity $\lambda$ set equal to unity. This also corresponds to the spacing between the wells. We later on scale all the lengths with respect to $\lambda$. The friction coefficient $\gamma$ is set to unity. $F(t)$ which corresponds to the externally applied time asymmetric force with zero average over the period is also shown in figure 1. The forces in the gentler and steeper sides of the potential are respectively $f^+ = -Q/\lambda_1$ and $f^- = Q/\lambda_2$ and $Q$ is the height of the potential. In the above expression we have also included the presence of an external load $L$, which is essential for defining thermodynamic efficiency. Following Stratonovich's interpretation [33], the corresponding Fokker–Planck equation [34] is given by

$$\frac{\partial P(q,t)}{\partial t} = \frac{\partial}{\partial q}\left[k_B T\frac{\partial P(q,t)}{\partial q} + (V'(q) - F(t) + L)P(q,t)\right]. \tag{2}$$

Since we are interested in the adiabatic limit we first obtain an expression for the probability current density $j$ in the presence of a constant external force $F$. The expression for the current [32] is

$$j(F) = \frac{{P_2}^2 \sinh[\lambda(F-L)/2k_B T]}{k_B T(\lambda/Q)^2 P_3 - (\lambda/Q)P_1 P_2 \sinh[\lambda(F-L)/2k_B T]} \tag{3}$$

where

$$P_1 = \Delta + \frac{\lambda^2 - \Delta^2}{4}\frac{F-L}{Q} \tag{4}$$

$$P_2 = \left(1 - \frac{\Delta(F-L)}{2Q}\right)^2 - \left(\frac{\lambda(F-L)}{2Q}\right)^2 \tag{5}$$

$$P_3 = \cosh[\{Q - 0.5\Delta(F-L)\}/k_B T] - \cosh[\lambda(F-L)/2k_B T] \tag{6}$$

where $\lambda = \lambda_1 + \lambda_2$ and $\Delta = \lambda_1 - \lambda_2$, the spatial asymmetry parameter. The current in the stationary regime averaged over the period $\tau$ of the driving force $F(t)$ is given by

$$\langle j\rangle = \frac{1}{\tau}\int_0^\tau j(F(t))\,dt. \tag{7}$$

We assume that $F(t)$ changes slowly enough, i.e., its frequency is smaller than any other frequency related to the relaxation rate in the problem such that the system is in a steady state at each instant of time.

In the present work we consider time asymmetric ratchets with a zero mean periodic driving force [15, 17, 19] given by

$$F(t) = \begin{cases} \dfrac{1+\epsilon}{1-\epsilon}F_0, & \left\{n\tau \le t < n\tau + \dfrac{1}{2}\tau(1-\epsilon)\right\}, \\ -F_0, & \{n\tau + \frac{1}{2}\tau(1-\epsilon) < t \le (n+1)\tau\}. \end{cases} \tag{8}$$

J. Stat. Mech. (2005) P04012

Here, the parameter $\epsilon$ signifies the temporal asymmetry in the periodic forcing, $\tau$ the period of the driving force $F(t)$ and $n = 0, 1, 2, \ldots$ is an integer. For this forcing in the adiabatic limit the expression for the time averaged current is [9, 17]

$$\langle j \rangle = j^+ + j^-, \tag{9}$$

with

$$\begin{aligned} j^+ &= \frac{1}{2}(1-\epsilon)\, j\left(\frac{1+\epsilon}{1-\epsilon}F_0\right), \\ j^- &= \tfrac{1}{2}(1+\epsilon)\, j(-F_0) \end{aligned} \tag{10}$$

where $j^+$ is the current fraction in the positive direction over a fraction of time period $(1-\epsilon)/2$ of $\tau$ when the external driving force field is $((1+\epsilon)/(1-\epsilon))F_0$ and $j^-$ is the current fraction over the time period $(1+\epsilon)/2$ of $\tau$ when the external driving force field is $-F_0$. The input energy $E_{\text{in}}$ per unit time is given by [9, 15]

$$E_{\text{in}} = F_0\left[\left(\frac{1+\epsilon}{1-\epsilon}\right) j^+ - j^-\right]. \tag{11}$$

It may be noted that the expression for input energy as given by Ai *et al* [19] is

$$E_{\text{in}} = F_0[j^+ - j^-]. \tag{12}$$

In our subsequent discussions we show that this expression for input energy, when used, leads to an efficiency value greater than 1, which is distinctly unphysical.

In order for the system to do useful work, a load $L$ is applied in a direction opposite to the direction of current in the ratchet. The overall potential is then $V(q) = [V_0(q) + qL]$. As long as the load is less than the stopping force $L_{\text{s}}$, current flows against the load and the ratchet does work. Beyond the stopping force the current flows in the same direction as the load application and hence no useful work is done. Thus, in the operating range of the load, $0 < L < L_{\text{s}}$, the Brownian particles move in the direction opposite to the load and the ratchet does useful work (storing energy in the form of a potential or, say, charging the battery). The average work done over a period is given by [9]

$$E_{\text{out}} = L[j^+ + j^-]. \tag{13}$$

The thermodynamic efficiency of energy transduction is [7, 8]

$$\eta = \frac{E_{\text{out}}}{E_{\text{in}}} = \frac{L[j^+ + j^-]}{F_0[((1+\epsilon)/(1-\epsilon))j^+ - j^-]}. \tag{14}$$

In the limit when the current fraction in the forward direction is much larger than that in the backward direction, $j^+ \gg j^-$, and at very low temperature (temperature tending to zero) the efficiency is given by [15] as

$$\eta = \frac{L(1-\epsilon)}{F_0(1+\epsilon)}. \tag{15}$$

In the same limit, using equation (12) of Ai *et al*, we get $\eta = L/F$ which is independent of $\epsilon$.

The suppression of backward current at low temperature occurs for values of $F_0$ less than $Q/\lambda_2$. However, finite current fraction flows in the positive direction when

$((1+\epsilon)/(1-\epsilon))F_0 > -Q/\lambda_1$ or $F_0 > Q(1-\epsilon)/\lambda_1(1+\epsilon)$. Hence, in the operating range of $F_0$, $Q/\lambda_2 > F_0 > Q(1-\epsilon)/\lambda_1(1+\epsilon)$, a high efficiency is expected in the low temperature regime [11].

In the absence of a load the particle moves in a periodic potential without tilting and hence the system does not store any energy. Consequently all the input energy in the steady state is dissipated away. In such a case the energy loss in the medium $E_{\rm L} = E_{\rm in}$. $E_{\rm L}$ in turn is equal to the heat $Q_{\rm h}$ transferred to the bath and thus the entropy production $S_{\rm p} = Q_{\rm h}/T = E_{\rm L}/T$ [8]. Thus the total increase in the entropy (or the entropy production) of the bath (universe) integrated over the period of the external drive is given by [8]

$$S_{\rm p} = \frac{Q_{\rm h}}{T} = \frac{E_{\rm in}}{T} = \frac{E_{\rm L}}{T}.$$

As discussed in the introduction, currents in ratchet systems are generated at the expense of entropy and thus we expect a correlation between the magnitude of the current and the total entropy production.

In our work all the physical quantities are taken in dimensionless units. Moreover, the energies and lengths are scaled with respect to $Q$, the barrier height, and $\lambda$, the spatial period of the potential, respectively. In the following section we present our results and discuss our calculations.

## 3. Results and discussion

We study the motion of an overdamped Brownian particle subjected to a time asymmetric periodic forcing but in the presence of a sawtooth potential. We present noticeable differences between the motion in a smooth potential as in [15] and that in a piecewise linear sawtooth potential. The role of smoothness or regularity in the potential in the efficiency of the energy transduction is clearly shown here.

To start with, in figure 2 we study the behaviour of efficiency with load in a spatially *symmetric* sawtooth potential ($\Delta = 0$) in the presence of a time asymmetric driving field for fixed values of $F_0 = 0.1$, $T = 0.01$ and $Q = 1$ for different values of $\epsilon$. Currents in this ratchet model arise solely due to the temporal asymmetry factor. For a given $\epsilon$, the efficiency increases as a function of load and then decreases. The value of efficiency attained is much higher than those attained in other models and it keeps increasing with increasing $\epsilon$. The stopping force $L_{\rm s}$ is also found to increase with increase in $\epsilon$. The larger $\epsilon$, the larger the current and efficiency as long as $F_0$ is less than the critical field, so that the barriers to motion in one direction alone disappear and there will be no current in the opposite direction. We notice that the efficiency depends linearly on the load as long as $L$ is much less than $L_{\rm s}$ where the backward motion is suppressed and the slope is given by $((1-\epsilon)/(1+\epsilon))F_0$ consistently with equation (15). In contrast to the case of a smooth sinusoidal potential [15] the locus of the peak in efficiency increases monotonically in the sawtooth case. The value of the efficiency is also much higher than that obtained in the smooth potential case. The input energy, $E_{\rm in}$, the output energy, $E_{\rm out}$, the fraction of currents $j^+$, $j^-$ and the average current $\langle j \rangle$ show the same qualitative behaviour as a function of load as is seen in figure 3 of [15] and the observed behaviour has been discussed in detail in [15]. Hence we do not deal with these quantities separately in the present work.

As noted in figure 2 one can attain an efficiency of the order of 40% for given physical parameters for the spatially symmetric ($\Delta = 0.0$) rocked ratchet. We now explore the

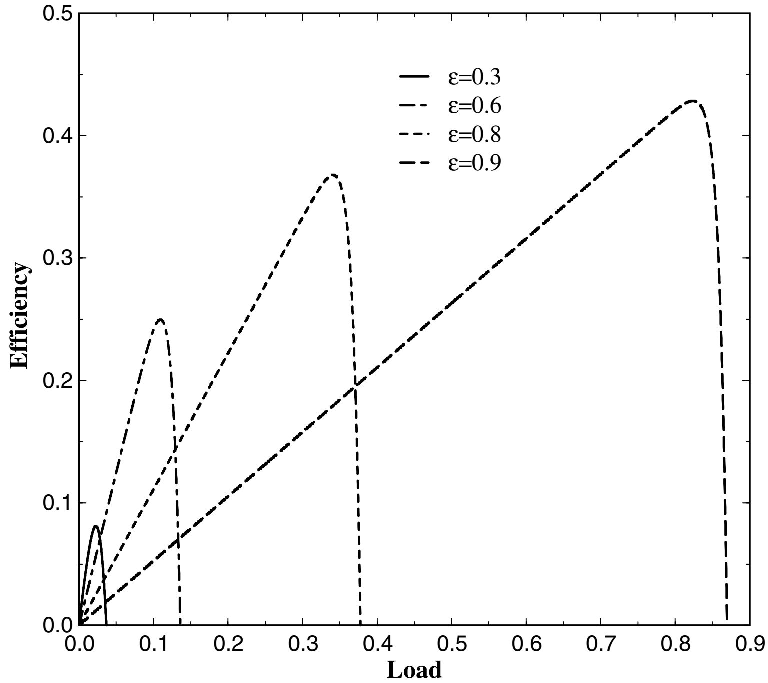


**Figure 2.** Efficiency versus load for $\Delta = 0.0, F_0 = 0.1,\ T = 0.01, Q = 1$ with varying $\epsilon$.

additional role of spatial asymmetry in the above results. For that, in figure 3 we plot efficiency as a function of load for various asymmetries in potential ($\Delta$) with fixed $F_0 = 0.1$, $T = 0.01$, $Q = 1$ and $\epsilon = 0.7$. We observe that an asymmetry in the potential enhances the efficiency and also increases the range of operation of the ratchet. As in the smooth potential case, the higher $\epsilon$, the larger the current and hence a larger load is necessary for the current to reverse its direction. From this figure it is clear that we can obtain a peak value of the efficiency of the order of 30% even in the absence of spatial asymmetry. This peak value of the efficiency and the range of operation of the load increase for higher asymmetry. For $\Delta = 0.9$ we obtain a peak value of the efficiency of more than 80% which is very high given the fact that the ratchet operates in an irreversible mode. It should also be noted that the initial slope of the efficiency versus load curve (for $L < L_s$) is the same, i.e., independent of $\Delta$, again consistent with equation (15). We can conclude from the above figure that additional spatial asymmetry will further help in enhancing the efficiency of time asymmetric ratchets. This is also due to the fact that the spatial asymmetry factor $\Delta$ is finite and positive and hence it enhances the currents in the system as compared to the case when $\Delta = 0.0$. Opposite conclusions will be reached on the effect of $\Delta$ on efficiency if $\Delta$ is negative, which is obvious. The reduction in currents when $\Delta$ is negative and $\epsilon$ is positive will be discussed in detail later in connection with current reversals.

In the inset of figure 3 we plot efficiency as a function of load for a representative positive and negative value of $\epsilon$ and $\Delta$ respectively. Here, one can clearly note that the attained efficiency is in the subpercentage regime. Our further analysis will be restricted to the case wherein $\Delta$ and $\epsilon$ remain positive as in this parameter space we naturally expect high efficiency.

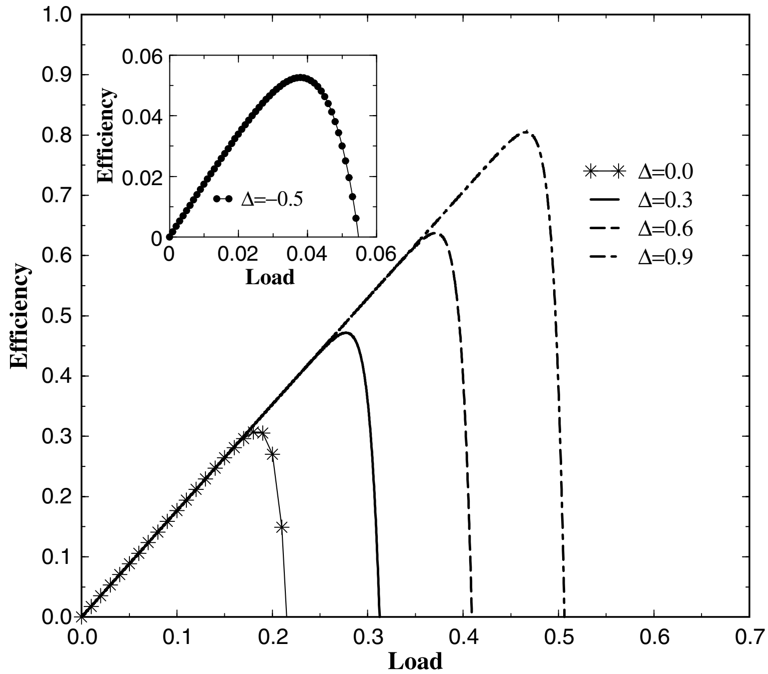


**Figure 3.** Efficiency versus load for various $\Delta = 0.9, 0.6, 0.3, 0.0$ with fixed $F_0 = 0.1$ $\epsilon = 0.7$, $T = 0.01$ and $Q = 1$. The inset shows the efficiency for $\Delta = -0.5$ with other parameters remaining the same.

We now study the role of the temporal asymmetry parameter $\epsilon$ for the case of spatially asymmetric ($\Delta = 0.9$) ratchets. Figure 4 shows the behaviour of efficiency as a function of load for varying $\epsilon$ for a fixed value of $\Delta = 0.9$. It is clear that the inclusion of time asymmetry leads to an enhanced value of the efficiency and the operational range of load. An efficiency of about $\sim$90% is readily attained as can be seen in figure 4. The locus of the peak value in efficiency monotonically increases with increase in $\epsilon$. This is in contrast to the non-monotonic behaviour observed in a smooth sinusoidal potential [15]. Moreover, the efficiencies are much higher for these ratchets with a discontinuous potential. For the case $\epsilon = 0$ we get an efficiency of $\sim$40%. Such a case with $\epsilon = 0$ and finite $\Delta$ is discussed in [11]. As has been mentioned earlier, the initial slopes are linear in accordance with equation (15). There are some studies in the deterministic limit where one can attain efficiency to the ideal limit ($\eta = 1$). However, these ratchets work in a reversible quasi-static mode of operation and not in the adiabatic regime [8, 11]. The protocols of the operation rely on synchronizing the dynamics of the particle with the external force [8, 11].

It has to be noted that our results cannot be compared with that of Ai *et al*, given in [19]. The inset of figure 4 shows a plot of efficiency as a function of load using the incorrect expression for the input energy used by Ai *et al*. Compared to our expression for efficiency in the limiting case, equation (15), the efficiency in the case of Ai *et al* is $\eta = L/F_0$ which is independent of $\epsilon$. This is clear from the initial slopes of the plots in the inset of figure 4. The important fact to be noted is that the efficiency value in some parameter ranges (as shown in figure 4) becomes larger than 1, which is clearly unphysical (i.e., $E_{\rm out} > E_{\rm in}$), when using the incorrect expression of Ai *et al* [19] for the input energy.

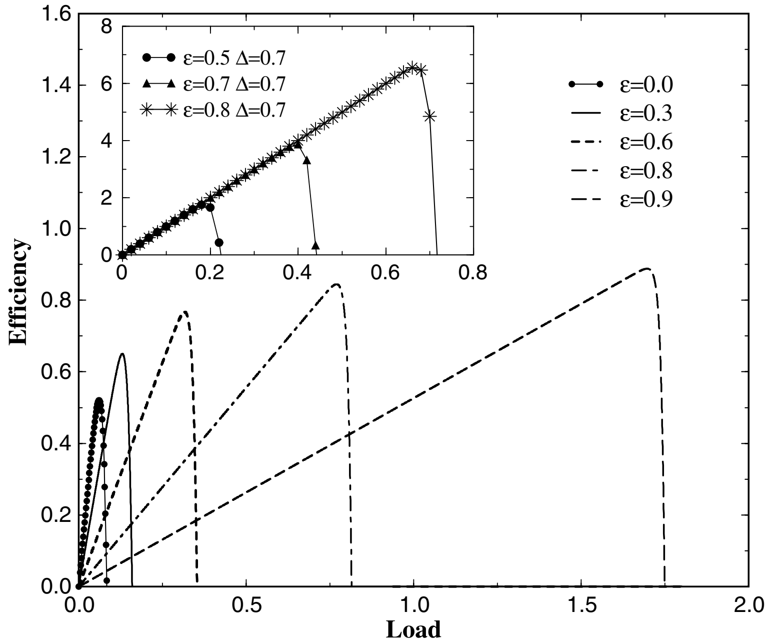


**Figure 4.** Efficiency versus load for $\Delta = 0.9$, $F_0 = 0.1$, $T = 0.01$ and $Q = 1$ with varying $\epsilon$. The inset shows the plot of efficiency versus load for $\Delta = 0.7$, $F_0 = 0.1$ and $T = 0.01$ for $\epsilon = 0.5, 0.7$ and 0.8 using the incorrect expression for input energy used by Ai *et al*, in [19].



Henceforth we confine our discussion to consideration using the corrected expression for the input energy.

In figure 5 we plot the efficiency as a function of $\epsilon$ for different strengths of potential asymmetry for $F_0 = 0.1$, $L = 0.1$, $Q = 1$ and $T = 0.01$. Similarly to in the earlier figure, the potential asymmetry is seen to increase the efficiency value. The larger the asymmetry in potential, the lower the value of $\epsilon$ for which one gets higher efficiency. This follows from the fact that the larger $\Delta$, the smaller the critical value of $\epsilon$ for getting current in the forward direction. The critical value of $\epsilon$, $\epsilon_c$, in the absence of load, is given by $\epsilon_c = (Q_0 - F_0\lambda)/(Q_0 + F_0\lambda)$. One can note that this critical value decreases as $F_0$ increases. In the absence of load the current vanishes for $\epsilon = 0.0$ and moreover the current fraction in the positive direction $j^+$ vanishes as $\epsilon \to 1$. Hence naturally a peak in efficiency as a function of $\epsilon$ is expected. For higher values of $\epsilon$ in the regime where the backward current is suppressed the slope in the figure is consistent with equation (15) (which is again independent of $\Delta$, as clearly seen in the figure).

In figure 6 we plot efficiency as a function of $F_0$ for the case of a symmetric potential ($\Delta = 0.0$) for different values of $\epsilon$ with fixed $L = 0.1$, $Q = 1$ and $T = 0.01$. Consistent with the general observation of this problem, for lower $\epsilon$ values we need larger $F_0$ to get forward current. Moreover, in the absence of load, the current vanishes in both limits of zero $F_0$ and large $F_0$. In the large $F_0$ limit, the barriers to motion in the forward as well as backward direction disappear and consequently the average current over the period vanishes. Thus a peak in the efficiency as a function of $F_0$ is obvious. Additional spatial asymmetry enhances the efficiency by a large amount. This can be clearly seen in figure 7

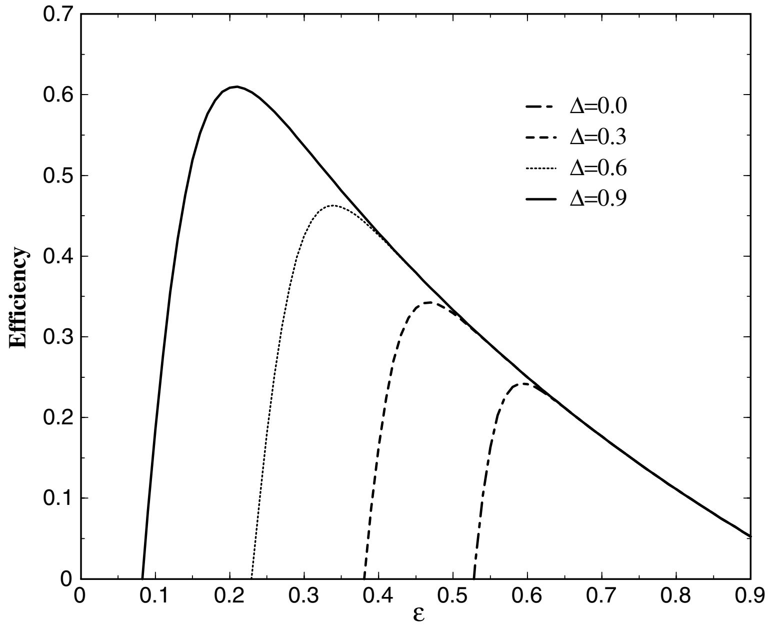


**Figure 5.** Efficiency versus $\epsilon$ for various values of $\Delta$ with fixed $F_0 = 0.1$, $Q = 1$, $L = 0.1$ and $T = 0.01$.

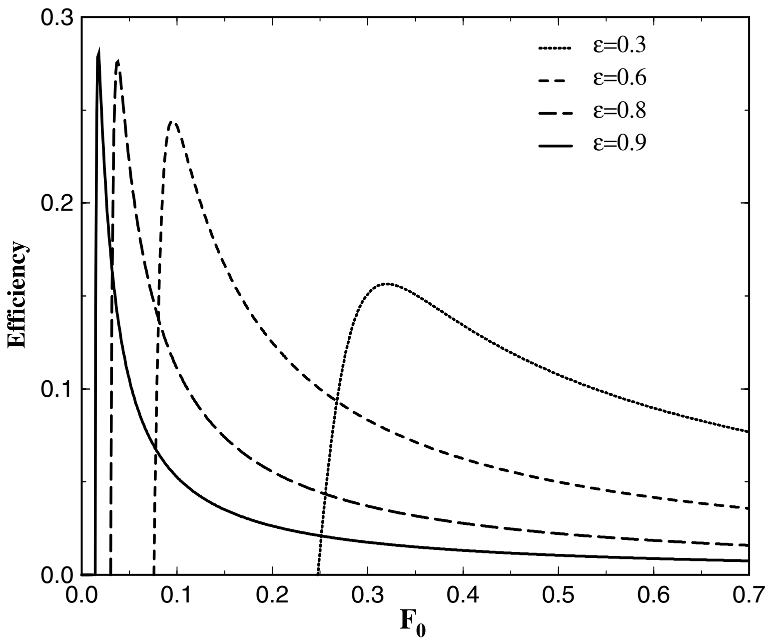


**Figure 6.** Efficiency versus $F_0$ for various values of $\epsilon$ for the symmetric case with fixed $L = 0.1$, $Q = 1$ and $T = 0.01$.

where we have plotted efficiency versus $F_0$ for the case $\Delta = 1.0$. The difference between figures 6 and 7 is that the envelopes of the peak values of the efficiency show opposite behaviour. In the case of a smooth potential we had observed earlier [15] that the envelope of the peak of efficiency decreases with increase in $\epsilon$ in contrast with that in figure 6.



J. Stat. Mech. (2005) P04012

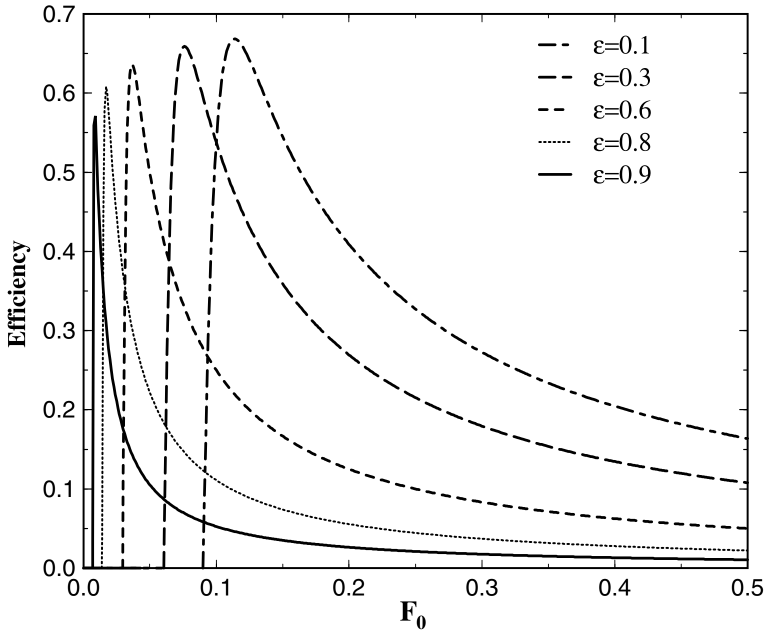


**Figure 7.** Efficiency versus $F_0$ for various values of $\epsilon$ with fixed $L = 0.1$, $Q = 1$, $\Delta = 1$ and $T = 0.01$.

So far we have shown that a large efficiency of the order of unity can be obtained readily in the time asymmetric rocked ratchets in the presence of additional spatial asymmetry. Notably, this large efficiency is obtained in the irreversible mode of operation in the adiabatic regime. In the presence of both $\Delta$ and $\epsilon$ we do not have to fine-tune the parameters and we get much higher efficiency above the subpercentage limit. In the following we address the question of whether thermal fluctuations (noise) can facilitate energy transduction, a subject which has been pursued widely and is of fundamental importance in its own right in the areas in which noise plays a constructive role [9].

In figure 8 we plot efficiency as a function of temperature for varying load and $\Delta$ with fixed $\epsilon = 0.9$. We observe that the efficiency decreases with noise strength ($T$). We find the value of the efficiency at very low temperature to exactly coincide with the values obtained from the analytical expression for efficiency in the limit $j^+ \gg j^-$, equation (15).

In figure 9 we plot efficiency as a function of temperature for different spatial asymmetry parameters $\Delta$ with fixed $L = 0.77$, $\epsilon = 0.9$, $Q = 1$ and $F_0 = 0.1$. One can note readily that at low temperatures the efficiency is independent of $\Delta$, equation (15), and it decreases with temperature. Also, as one increases $\Delta$ a larger range of temperature is obtained over which the efficiency value is high. In the parameter range we have considered we generally observe that temperature (noise) cannot facilitate energy transduction, i.e., it cannot optimize the efficiency. This is in spite of the fact that in all the cases current as a function of temperature exhibits a peaking behaviour (for example); see the insets of figures 8 and 9 for a particular representative parameter value mentioned in the figure captions.

However, with a judicial choice of parameters which require fine-tuning we obtain a regime in parameter space where the efficiency exhibits a peak with temperature. In

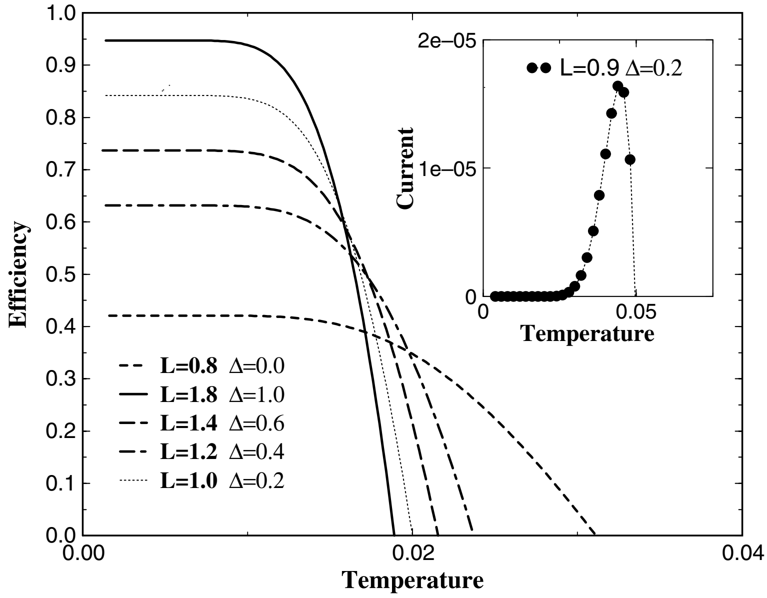


**Figure 8.** Efficiency versus temperature for various values of load and $\Delta$ with fixed $Q = 1$, $F_0 = 0.1$ and $\epsilon = 0.9$. The inset shows the peaking of the current with temperature for $\Delta = 0.2$, $F_0 = 0.1$, $L = 0.9$, $Q = 1.0$ and $\epsilon = 0.9$.

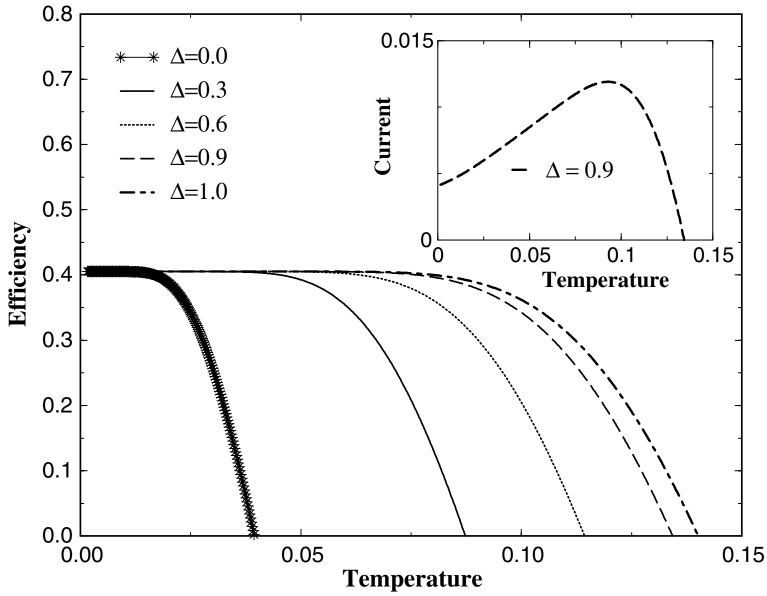


**Figure 9.** Efficiency versus temperature for various values of $\Delta$ with fixed $\epsilon = 0.9$, $F_0 = 0.1$, $L = 0.77$ and $Q = 1$.

this parameter range, temperature or noise facilitates energy transduction. Figure 10 shows the peaking behaviour of the thermodynamic efficiency with temperature for three representative sets of parameters mentioned in the figure caption. The magnitudes of the current and efficiency are, however, quite small in this range, which we have verified separately.

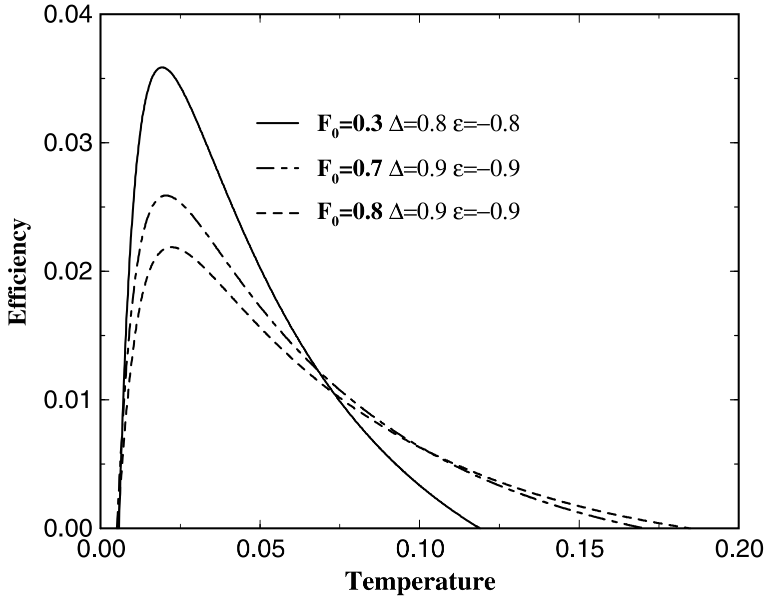


**Figure 10.** Efficiency versus temperature for three different cases of physical parameters. (i) $F_0 = 0.3$, $\Delta = 0.8$, $\epsilon = -0.8$; (ii) $F_0 = 0.7$, $\Delta = 0.9$, $\epsilon = -0.9$; (iii) $F_0 = 0.8$, $\Delta = 0.9$, $\epsilon = -0.9$ for fixed $L = 0.01$, and $Q = 1$.

To understand this behaviour of the efficiency with temperature, in figure 11 we plot the input energy ($E_{\text{in}}$) and the output work ($E_{\text{out}}$) as a function of temperature. The input energy is found to increase monotonically with temperature. However, the output energy shows a peak with temperature. The output energy curve is blown up by a factor of 1000 to make it comparable with the scale chosen. At very low temperature ($T < 0.006$) the efficiency is negative. The current in the absence of load is very small in this regime. For a given applied load, the current flows in the direction of the load and consequently the output energy is also negative (which could not be seen on the scale we have chosen in the figure). The output energy then increases with temperature and becomes positive for $T > 0.06$. At the crossover points the finite value of the input energy gives rise to zero efficiency, since the output work is zero. As the temperature is increased the output work increases non-monotonically and then becomes zero at a temperature value of about 0.21, beyond which (i.e., beyond the operating range of the load) the current flows in the direction of the load. Thus at $T \sim 0.21$ the output energy and consequently the efficiency is zero. Hence we expect a peaking behaviour in efficiency as a function of temperature as is shown in the inset of the figure. It should be noted that the current in the absence of load shows a peak with temperature.

In figure 12 we plot the input and output energies for the case where the efficiency monotonically decreases with temperature. All the physical parameters are mentioned in the figure captions. In contrast to the case observed in figure 11, we note that both the output energy and the input energy are finite at zero temperature leading in turn to a finite value of the efficiency. As we increase the temperature, the input energy increases monotonically whereas the output energy exhibits a small peak. Beyond a temperature of 0.52, $E_{\text{out}}$ becomes negative. The rise in input energy is very rapid as compared to



J. Stat. Mech. (2005) P04012

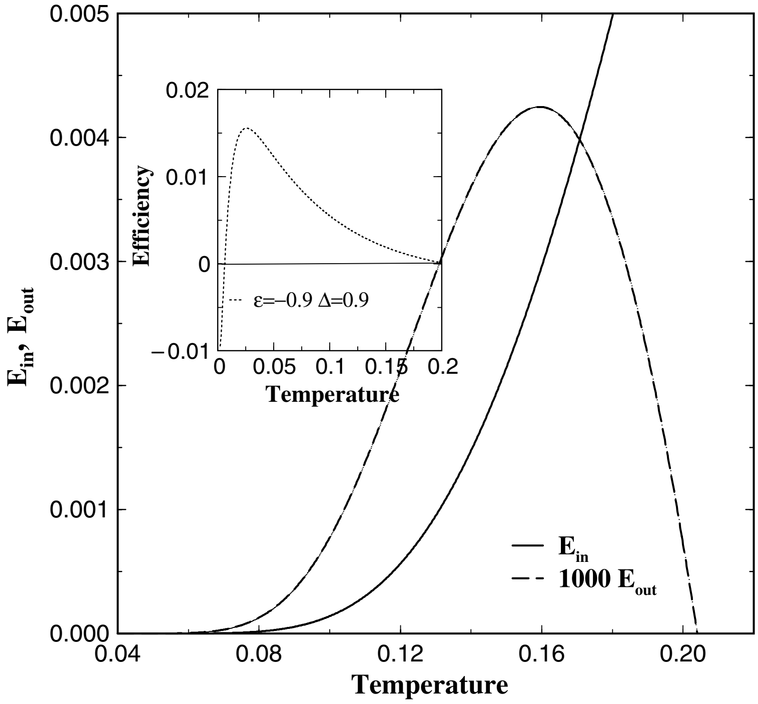


**Figure 11.** The input and output energy versus temperature for $\Delta = 0.9, F_0 = 0.10, L = 0.01, Q = 1$ and $\epsilon = -0.9$. The output energy curve is blown up by a factor of 1000 to make it comparable with the scale chosen. The inset shows the behaviour of the efficiency for the same set of parameters.

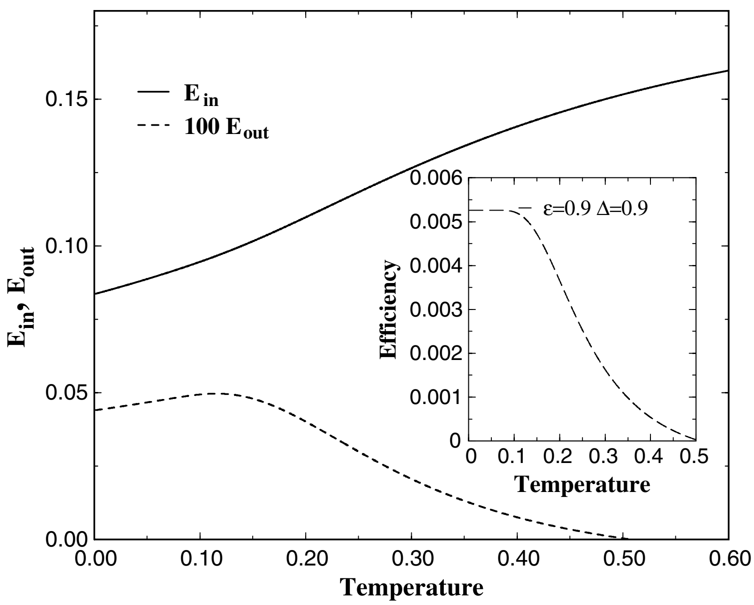


**Figure 12.** Input and output energy versus temperature for $\Delta = 0.9, F_0 = 0.1, L = 0.01, Q = 1$ and $\epsilon = 0.9$. The output energy curve is blown up by a factor of 100 to make it comparable with the scale chosen. The inset shows the behaviour of the efficiency with temperature for the same set of parameters.

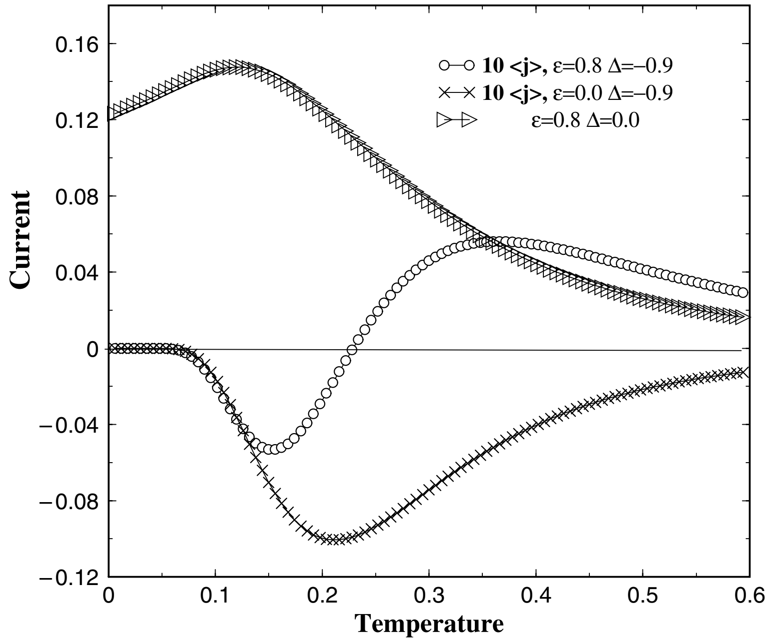


**Figure 13.** Current versus temperature for $\Delta = -0.9, F_0 = 0.3, Q = 1, L = 0$ and $\epsilon = 0.8$. $\langle j \rangle$ is multiplied by a factor of 10 for the cases (i) $\epsilon = 0.8$, $\Delta = -0.9$ and (ii) $\epsilon = 0.0$, $\Delta = -0.9$ to make it comparable with the scale chosen.

that of the output energy and consequently the efficiency decreases monotonically with temperature as shown in the inset of the figure up to $T = 0.52$, beyond which it becomes negative.

So far we have discussed the nature of the efficiency of energy transduction as a function of system variables. We now concentrate on another aspect of ratchet systems, namely, current reversals, which play a central role in designing separation devices. It is known that symmetrically rocked spatially asymmetric ratchets do not exhibit current reversals in the adiabatic regime [2, 35, 36]. However, the presence of system inhomogeneities (frictional or inhomogeneous ratchets) can induce single or multiple current reversals even in the adiabatic regime [15, 37]. In our present case of homogeneous ratchets it is easy to tune current reversals as there are two asymmetric parameters present in the problem. In figure 13 we plot current as a function of $T$ for a particular value of $\epsilon$ and $\Delta$ given in the caption. The parameters are chosen such that the direction of current in the presence of either of the parameters alone should be in opposite directions. For example, in figure 13 the current is in the positive direction when $\epsilon = 0.8$ and $\Delta = 0$ whereas it is in the reverse direction when $\epsilon = 0.0$ and $\Delta = -0.9$. So by tuning a combination of these two parameter values for $\epsilon = 0.8$ and $\Delta = -0.9$ one gets current reversal as a function of $T$.

It should be noted that this is not an additive effect arising separately from $\epsilon$ and $\Delta$. The current reversal arises due to the complex interplay of these two asymmetry parameters. It should be emphasized that once current inversion upon the variation of one parameter is established, an inversion upon variation of any other parameter can be readily inferred. For details we refer the reader to [2]. In accordance with the above

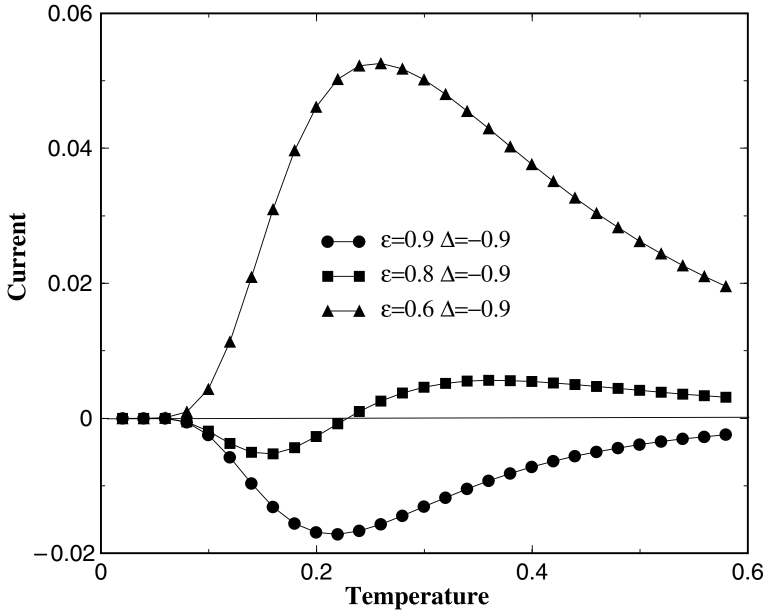


**Figure 14.** Current versus temperature for $\Delta = -0.9, F_0 = 0.3, Q = 1, L = 0$ and varying $\epsilon$.

reasoning for current reversals, in figure 14 we have plotted current versus temperature with a fixed value of $\Delta = -0.9$ and varying $\epsilon$. As we vary $\epsilon$ from a large value to a small value, in the intermediate range of $\epsilon$ we get current reversal.

Having discussed efficiency and the nature of currents and their reversals we now study other thermodynamic quantities, namely, the input energy and entropy production. We would like to find whether any relation exists among them and the nature of the currents as discussed in the introduction. Some recent studies have also attempted to reveal the relations between two completely unrelated phenomena, namely, stochastic resonance and Brownian ratchets, in a formal way, through the consideration of Fokker–Planck equations [38]. Stochastic resonance is a phenomenon where we can obtain optimal output from a system by adding noise to the system [27]. It has been argued that the rate of flow of particles in a Brownian ratchet is analogous to the rate of flow of information in the case of stochastic resonance [39].

In figure 15 we plot the entropy production, current and input energy for a representative case, $\Delta = 0.4$, $F_0 = 0.1$ and $\epsilon = 0.8$, as a function of temperature or noise strength. We observe that current exhibits a peak as a function of temperature while the input energy is a monotonically increasing function of temperature [28]. It has been argued earlier that the peak in the input energy is a good measure for the occurrence of stochastic resonance in the dynamics of the particle [29, 30]. It is natural to expect that at resonance the system will extract more input energy from the environment, which in turn is subsequently dissipated away in the steady state (for details see [29, 30]). However, the observed monotonic behaviour of the input energy, as opposed to the nature of the current, rules out the possibility of any resonance in the dynamics of the particle as a function of noise strength.

J. Stat. Mech. (2005) P04012

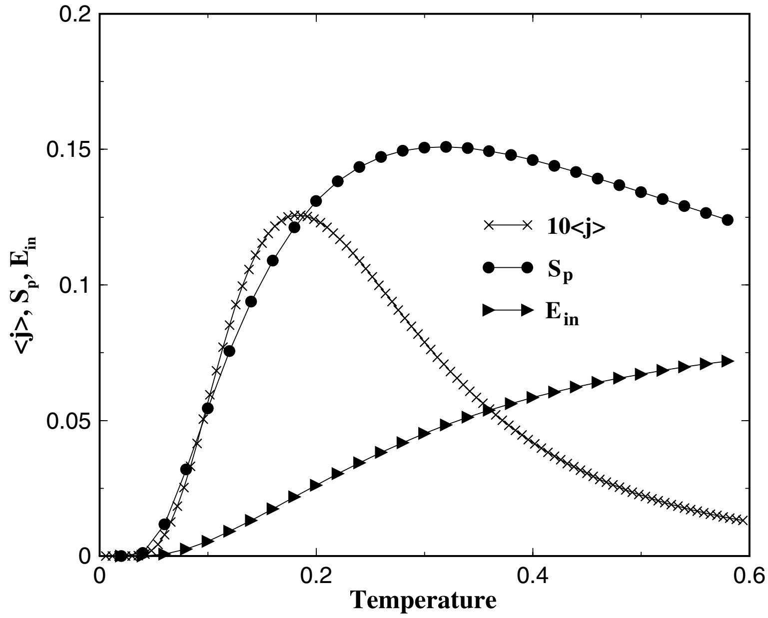


**Figure 15.** $\langle j \rangle$, $S_{\mathrm{p}}$ and $E_{\mathrm{in}}$ versus temperature for $\Delta = 0.4$ with fixed $F_0 = 0.1$, $Q = 1$, $L = 0$ and $\epsilon = 0.8$. The current $\langle j \rangle$ is blown up by a factor of ten to make it clearer.

It may be noted that stochastic resonance is also related to the synchronization phenomenon [40]. For example, in the context of the study of stochastic resonance in driven double-well systems, the input energy in the steady state is related to the dissipative or hysteresis losses in the system. This input energy loss (or the area of the hysteresis loop) in a driven double-well system has been shown to be a good measure of synchronization of the passages of the particle between two wells with respect to the applied field. For details see [40]. Our present results are valid only for the case of an adiabatically rocked thermal ratchet and hence does not rule out the possibility of resonance and synchronization in the dynamics of the particle in the nonadiabatic regime or in other ratchet systems.

The presence of net currents in the ratchet increases the amount of information known about the system than in its absence. This extra bit of information comes from the 'negentropy' or the physical information supplied by the external nonequilibrium bath. Since the currents are generated at the expense of entropy one normally expects the maxima in current and the maxima in the overall entropy production to occur at the same value of the noise strength. In fact, in a related development it has been pointed out that the amount of information transferred by the nonequilibrium bath is quantified in terms of algorithmic complexity. Moreover, the algorithmic complexity or Kolmogorov information entropy exhibits a maximum at the value of a physical parameter where the current is a maximum [41]. From figure 15, we see that the entropy production, $S_{\mathrm{p}}$, also exhibits a peak as a function of noise strength. The peaks in the average current, $\langle j \rangle$, and total entropy production, $S_{\mathrm{p}}$, do not occur at the same $T$. This clearly indicates that maxima in the entropy production do not arise at the value where the current is a maximum, thereby ruling out a correlation between the entropy production peak and the peak in the current [28].

## 4. Conclusions

We have studied in detail the nature of energetic efficiency driven by zero-average time asymmetric forcing in the adiabatic limit. The potential is taken to be of the sawtooth type characterized by an asymmetry parameter $\Delta$. We have shown that, in the presence of temporal and spatial asymmetry, a much higher efficiency, above the subpercentage regime (known for other ratchets), can be readily obtained. Spatial asymmetry together with temporal asymmetry gives larger efficiency as compared to the presence of spatial or temporal asymmetry alone. At low temperatures an efficiency value closer to the ideal limit can be obtained by judicious tuning of physical parameters even though the operation of the ratchet is in the irreversible mode. In a bigger range of parameter space temperature does not facilitate energy transduction. By fine-tuning the parameters one can obtain a regime in which temperature facilitates energy transduction. However, in this parameter space the value of the efficiency attained is found to be at the subpercentage level.

We also observe current reversals in the adiabatic limit by proper tuning of different parameters. These reversals are attributed to the complex dynamics of the system. From our study of the nature of the input energy and currents we conclude that there is no resonance phenomenon occurring in the system. The analysis of current and entropy production results shows that the peaks in current and entropy production do not coincide.

It is worthwhile exploring whether the transport in these efficient ratchets is coherent or not. Noise induced currents are always accompanied by a diffusive spread. If the diffusive spread of the particle is less than the average distance (say, the length of the period of the potential) travelled by the particle in a given time, then the transport is said to be coherent [42]. This is quantified in terms of the so-called dimensionless Peclet number. Recently it has been shown that fluctuation statistics of the noise induced current plays a crucial role in determining the rectification efficiency of Brownian motors. The smaller the value of the variance of the velocity fluctuations, the higher the rectification efficiency [43]. The present work concentrates mainly on optimizing the thermodynamic efficiency. One could also optimize maximum work or have the best compromise between maximum work and efficiency. This can be done using known optimizing criteria [12]. Studies in this regard are currently in progress.

## References

[1] Jülicher F, Ajdari A and Prost J, 1997 *Rev. Mod. Phys.* **69** 1269
[2] Reimann P, 2002 *Phys. Rep.* **361** 57 and references therein
[3] Jayannavar A M, 2001 *Preprint* cond-mat/0107079
Bhattacharjee J K and Chakrabarti B K (ed), 2005 *Indian J. Phys.* at press (Frontiers in Condensed Matter Physics, A commemorative volume to mark the 75th year)
[4] Linke H (ed), 2002 *Appl. Phys.* A **75** (2) (Special issue on Ratchets and Brownian motors: basics, experiments and applications)
[5] Hänggi P, Marchesoni F and Nori F, 2005 *Ann. Phys. Lpz.* **14** 51
[6] For details see http://seneca.fis.ucm.es/parr/GAMES/
[7] Sekimoto K, 1997 *J. Phys. Soc. Japan* **66** 6335
[8] Parrondo J M R and De Cisneros B J, 2002 *Appl. Phys.* A **75** 179
[9] Kamegawa H, Hondou T and Takagi F, 1998 *Phys. Rev. Lett.* **80** 5251
Takagi F and Hondou T, 1999 *Phys. Rev.* E **60** 4954
Sumithra K and Sintes T, 2001 *Physica* A **297** 1
Dan D and Jayannavar A M, 2002 *Phys. Rev.* E **65** 037105



J. Stat. Mech. (2005) P04012

[10] Krishnan R and Jayannavar A M, 2005 *Physica* A **345** 61
[11] Sokolov I M, 2001 *Phys. Rev.* E **63** 021107
Sokolov I M, 2002 *Preprint* cond-mat/0207685v1
[12] Sánchez Salas N and Hernández A C, 2003 *Phys. Rev.* E **68** 046125
[13] Makhnovskii Yu A, Rozenbaum V M, Yang D-Y, Lin S H and Tsong T Y, 2004 *Phys. Rev.* E **69** 021102
[14] Astumian R D, 1996 *J. Phys. Chem.* **100** 19075
[15] Krishnan R, Mangal C M and Jayannavar A M, 2004 *Phys. Rev.* E **70** 21102
[16] Ajdari A, Mukamel D, Peliti L and Prost J, 1994 *J. Physique* I **4** 1551
[17] Chialvo D R and Millonas M M, 1995 *Phys. Lett.* A **209** 26
[18] Mahato M C and Jayannavar A M, 1995 *Phys. Lett.* A **209** 21
[19] Ai B-Q, Wang X J, Liu G T, Wen D H, Xie H Z, Chen W and Liu L G, 2003 *Phys. Rev.* E **68** 061105
[20] Marchesoni F, 1986 *Phys. Lett.* A **119** 221
[21] Savel'ev S, Marchesoni F, Hänggi P and Nori F, 2004 *Europhys. Lett.* **67** 179
Savel'ev S, Marchesoni F, Hänggi P and Nori F, 2004 *Phys. Rev.* E **70** 066109
Savel'ev S, Marchesoni F, Hänggi P and Nori F, 2004 *Eur. Phys. J.* B **40** 403
[22] Chacon R and Quintero N R, 2005 *Preprint* physics/0503125 and references therein
[23] Savel'ev S, Marchesoni F and Nori F, 2003 *Phys. Rev. Lett.* **91** 010601
Savel'ev S, Marchesoni F and Nori F, 2004 *Phys. Rev. Lett.* **92** 160602
[24] Shmelev G M, Song N H and Tsurkan G I, 1985 *Sov. Phys. J. (USA)* **28** 161
Entin M V, 1989 *Sov. Phys. Semicond.* **23** 664
Atanasov R, Haché A, Hughes J L P and Sipe J E, 1996 *Phys. Rev. Lett.* **76** 1703
Haché A, Kostoulas Y, Atanasov R, Hughes J L P, Sipe J E and van Driel H M, 1997 *Phys. Rev. Lett.* **78** 306
Alekseev K N, Erementchouk M V and Kusmartsev F V, 1999 *Europhys. Lett.* **47** 595
[25] Schiavoni M, Sanchez-Palencia L, Renzoi F and Grynberg G, 2003 *Phys. Rev. Lett.* **90** 094101 and references therein
Jones P H, Goonasekera M and Renzoni F, 2004 *Phys. Rev. Lett.* **93** 073904
[26] Doering C R and Dontcheva L A, 1998 *Chaos* **8** 643
[27] Gammaitoni L, Hänggi P, Jung P and Marchesoni F, 1998 *Rev. Mod. Phys.* **70** 223
[28] Krishnan R, Mangal C M and Jayannavar A M, 2004 *Indian J. Phys.* **78** 747
[29] Iwai T, 2001 *Physica* A **300** 350
Iwai T, 2001 *J. Phys. Soc. Japan* **70** 353
[30] Dan D and Jayannavar A M, 2005 *Physica* A **345** 404
[31] Dan D, Mahato M C and Jayannavar A M, 1999 *Phys. Lett.* A **258** 217
Dan D, Mahato M C and Jayannavar A M, 2000 *Int. J. Mod. Phys.* B **14** 1585
[32] Magnasco M O, 1993 *Phys. Rev. Lett.* **71** 1477
[33] Stratonovich R L, 1958 *Radiotekh. Elektron. (Moscow)* **3** 497
Kuznetsov P I, Stratonovich R L and Tikhonov V I (ed), 1965 *Non-Linear Transformations of Stochastic Processes* (Oxford: Pergamon) (translation)
[34] Risken H, 1984 *The Fokker–Planck Equation* (Berlin: Springer)
[35] Dan D, Mahato M C and Jayannavar A M, 2001 *Phys. Rev.* E **63** 056307
[36] Reimann P, Bartussek R, Häussler R and Hänggi P, 1996 *Phys. Lett.* A **215** 26
[37] Dan D and Jayannavar A M, 2000 *Int. J. Mod. Phys.* B **14** 1585
Dan D, Mahato M C and Jayannavar A M, 1999 *Phys. Rev.* E **60** 6421
[38] Bier M, 1997 *Stochastic Dynamics* vol 386 (Berlin: Springer) pp 81–7
Berdichevsky V and Gitterman M, 1998 *Physica* A **249** 88
Astumian R D and Moss F, 1998 *Chaos* **8** 533
Qian M, Wang Y and Ahang X-J, 2003 *Chin. Phys. Lett.* **20** 810
[39] Allison A and Abbott D, 2001 *Fluct. Noise Lett.* **1** L239
[40] Mangal C M and Jayannavar A M, 1998 *Physica* A **248** 138
[41] Sanchez J R, Family F and Arizmendi C M, 1998 *Phys. Lett.* A **249** 281
[42] Lindner B and Schimansky-Geier L, 2002 *Phys. Rev. Lett.* **89** 230602
Dan D and Jayannavar A M, 2002 *Phys. Rev.* E **66** 041106
Krishnan R, Dan D and Jayannavar A M, 2005 *Physica* A at press [cond-mat/0501106]
[43] Machura L, Kostur M, Talkner P, Łuczka J, Marchesoni F and Hänggi P, 2004 *Phys. Rev.* E **70** 061105
Suzuki D and Munakata T, 2003 *Phys. Rev.* E **68** 021906

J. Stat. Mech. (2005) P04012