QCD phase transition inside Neutron stars: Role of Magnetic field

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Outline

- Introduction
- Phase transition inside Neutron stars
- Magnetic effect
- Summary and discussion

Quark Matter

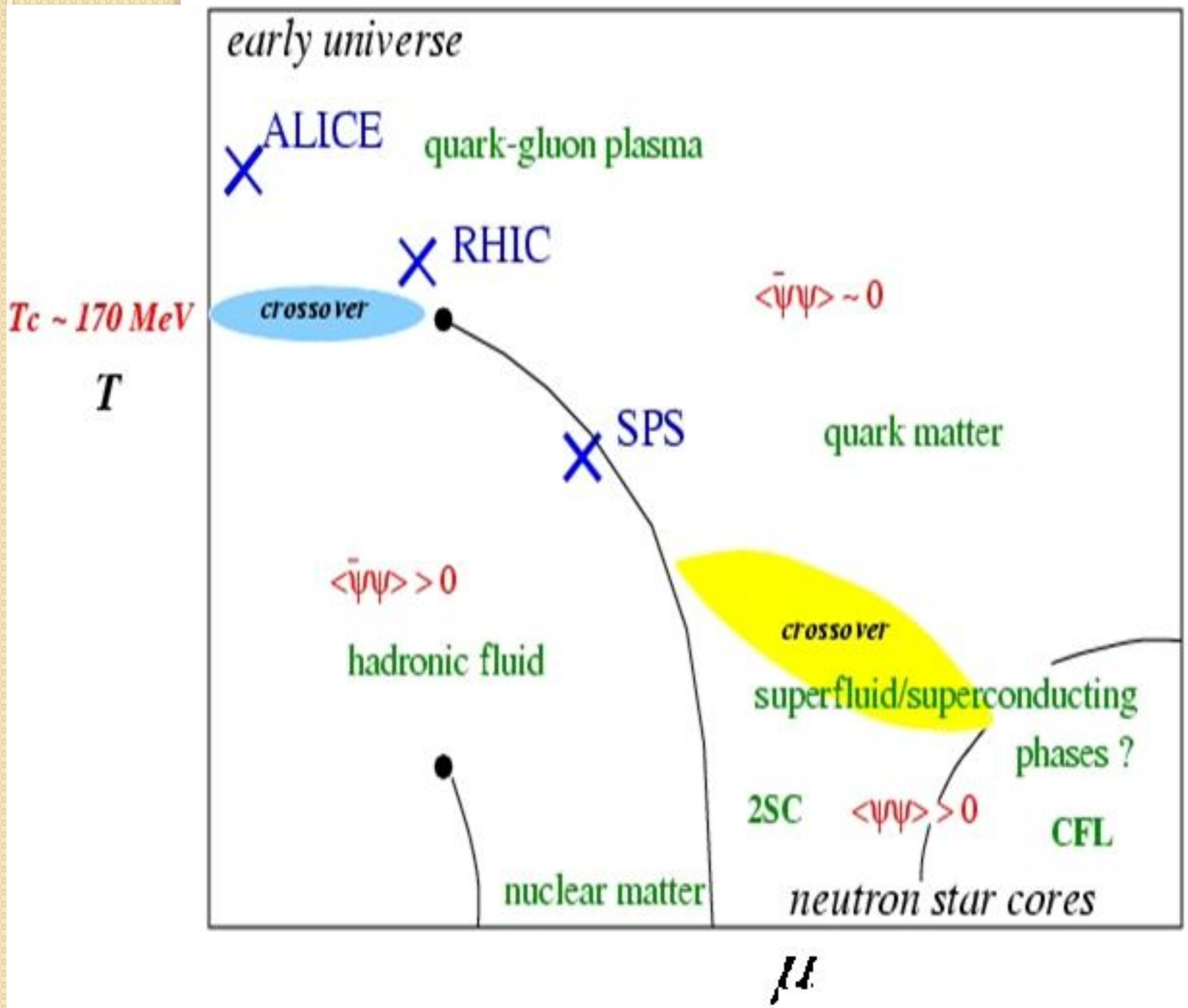
T >> μ

- Quark-gluon plasma phase
- filled the universe after microsecond of big bang
- created in the laboratory in relativistic heavy ion collisions

T << μ

- rich variety of symmetry breaking
- might exist in the interior of compact objects like neutron stars
- time enough for weak interaction – strangeness production

Introduction



Witten conjecture : (**1984 seminal paper PRD 30, 272**) **Strange quark matter (SQM) the true ground state of strongly interacting matter at high density and/or temperature.**

SQM : Almost equal numbers of up (u), down (d) and strange (s) quarks.

Quark Matter and neutron stars

- Different facets:

(a) Symmetry structures at high density systems

(b) Consequences of a quark matter phase
for example : gravitational waves
GRB connection

(c) Mechanism of the phase transition itself

Rotating Neutron Star:

The EOS obtained is used to solve Einstein's equation for rotating star.

The metric for the rotating star is

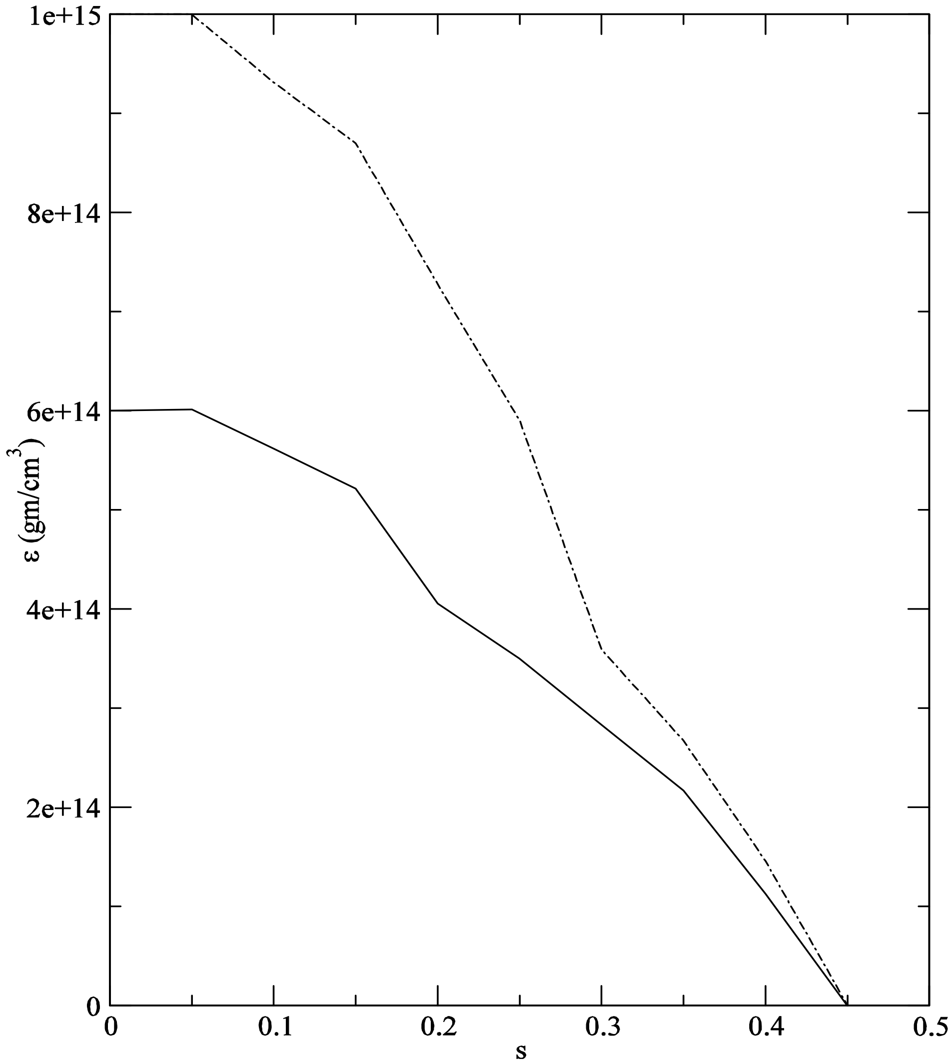
$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2) + e^{\gamma-\rho}r^2 \sin^2\theta(d\phi - \omega dt)^2,$$

Physical quantities are calculated from these potentials.

Solution of the potentials and the calculation of physical quantities is done using "RNS" code.

where $\alpha, \gamma, \rho,$ and ω are the gravitational potentials

Density Profile :



R/R_e = s/(1-s)

R: radius

R_e = equatorial radius

$R/Re = s/(1-s)$

Equator $s = 0.5$

AB, SKG, SR
PL B635 (2006) 195

Mechanism of phase transition

- **Two step process**

(a) hadronic matter to two flavour matter

- deconfinement
- t ~ milliseconds

(b) two flavour to three flavour matter

- generation of strange quark via weak process
- t ~ 100 seconds

Presence of two fronts ?

Bhattacharyya et al. PRC74, 065804 (2006)

- **General relativistic effects**

Bhattacharyya et al. PRC76, 052801(R) (2007)

Two step process

Conversion being a two step process (PRC 74, 065804, 2006).

First step : Deconfinement of nuclear matter to 2-flavour (u & d) quark matter. Strong interaction time scale.

Second step : Generation of strange quark from excess down quark. Weak interaction process.

The conversion to 2-flavour

Equation of state (EOS)
Nonlinear Walecka model for nuclear matter EOS.
Quark matter EOS determined by enforcing baryon number conservation.

Assumptions
The neutron star is spherically symmetric.
Existence of a combustion front separating the two phases.
Front generates from the centre and propagates outwards.

Results

Time taken by the front to reach the surface is of the order of few ms.

Time needed for the conversion is of the order of 100 s.

General relativistic approach

Actual calculation of the propagation of the front should involve GR, taking into account the curvature of the front. (*PRC 76 ®, 052801, 2007*)

Metric of the star structure

$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2)$$

$$+ e^{\gamma-\rho} r^2 \sin^2\theta(d\phi - \omega dt)^2$$

Metric is solved using the RNS code.
Code is solved for static and rotating star.
Rotating star gets deformed to an oblate spheroid shape.

General relativistic approach

Central density is 6 times nuclear matter saturation density.
Initial velocity of the front is 0.4

Results

The GR effect increases the velocity considerably. Most pronounced for static star.
The rotational effect suppress the GR effect.

General relativistic approach

Results

The velocity is maximum along the pole and minimum along the equator.

At any instant it may so happen that the polar region gets converted whereas along the equator the front is still propagating.

At some distance from the centre, the front breaks up into several distinct front and propagates with different velocities along different directions.

General relativistic approach

Results

The total time taken for the conversion is of the order of few ms.

The static star takes less time than the rotating star owing to the enlarged equatorial radius.

Compression at the pole assures that it takes minimum time.

Magnetic effect

Model predicted only the existence of bare quark star. Observational evidence points to the fact of existence of hybrid stars.

No damping term in the differential equation to stop the front.

Important property of neutron star
High magnetic field

Magnetic effect

Range of observed surface magnetic field is as high as 10^8-10^{12} G

Low mass X-ray binaries and ms pulsars have weaker fields 10^8-10^9 G

Young classical pulsars and high mass X-ray binaries have high fields 10^{10}-10^{12} G

Canonical picture of the classical pulsar mechanism involves (Michel 1982) a magnetic dipole at the center of a rotating NS.

Origin of magnetic field

Flux conservation

generation of magnetic fluxes earlier in progenitor stars and then its subsequent trapping inside the compact objects. But the fields inside a main sequence progenitor may not be enough to yield the needed magnetic field inside a neutron star as the neutron star contains only aound 15% of the progenitor mass (Spruit) 2008

(Rudderman 1972; Reisenegger 2001; Ferrario & Wickramasinghe 2005a,b, 2006) .

may also be generated through **dynamo processes** in a rotating and convecting fluid. (Thompson & Duncan 1993) newborn neutron stars are likely to combine convection and differential rotation making it favorable for dynamo process to operate. It was also suggested that the fields generated could explain the objects like soft gamma repeater and anomalous X-ray pulsars (Duncan & Thompson 1992; Thompson & Duncan 1995, 1996).

In order to study the effect of magnetic field on NS characteristics, one should really model the equilibrium configurations of NS in the framework of General Theory of relativity including both toroidal and poloidal magnetic field. However, in the present work our main aim is to understand the effects that magnetic field will have on the conversion front. So, to this end, we have borrowed some realistic magnetic field configurations existing in the literature as described below.

Equation of continuiy

$$\frac{1}{\varpi}\left(\frac{\partial\epsilon}{\partial\tau}+v\frac{\partial\epsilon}{\partial r}\right)+\frac{1}{W^2}\left(\frac{\partial v}{\partial r}+v\frac{\partial v}{\partial\tau}\right)+\frac{2v}{r}+\frac{v}{r}cot\theta=-v\left(\frac{\partial\gamma}{\partial r}+\frac{\partial\alpha}{\partial r}\right)$$

Euler equation

$$\frac{1}{\varpi}\left(\frac{\partial p}{\partial r}+v\frac{\partial p}{\partial\tau}\right)+\frac{1}{W^2}\left(\frac{\partial v}{\partial\tau}+v\frac{\partial v}{\partial r}\right)-\frac{1}{\varpi}\left(\vec{j}\times\vec{B}\right)=\frac{1}{2}\left(A\frac{\partial\gamma}{\partial r}+B\frac{\partial\rho}{\partial r}+C\frac{\partial\omega}{\partial r}+E\right)$$

The Lorentz force can be written as

$$\begin{aligned}\vec{j}\times\vec{B} &= \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B})\times\vec{B}\\ &= \frac{1}{\mu_0}(\vec{B}.\vec{\nabla})\vec{B}-\vec{\nabla}\left(\frac{B^2}{2\mu_0}\right)\end{aligned}$$

Static Star

First we perform our calculation for the static star considering both poloidal and toroidal field configuration following Colaiuda et al. (2008), for the field extending throughout the star

$$\overrightarrow{B} = \overrightarrow{B}(B_r, B_\theta, B_\phi)$$

$$B_r = -2B_0 e^{\alpha} cos\theta$$

$$B_\theta = 2B_0 e^{-\alpha} r sin\theta$$

$$B_\phi = \zeta B_0 e^{-(\gamma+\rho)/2} r^2 sin^2\theta$$

where ζ provides the relative weight factor for the contribution of toroidal field. The force is purely poloidal if $\zeta = 0$, and both poloidal and toroidal contribute if $\zeta = 1$. For both the cases the hydrodynamic equation gets modified due to the inclusion of the magnetic field.

$$\frac{dr}{d\tau} = vG \qquad G = \frac{e^{(\gamma+\rho)/2}}{e^{\alpha}}$$

$$\frac{\partial v}{\partial r} = \frac{W^2 v[K + K1 + K2]}{2[v^2(1+G)^2 - n(1+v^2G)^2]}.$$

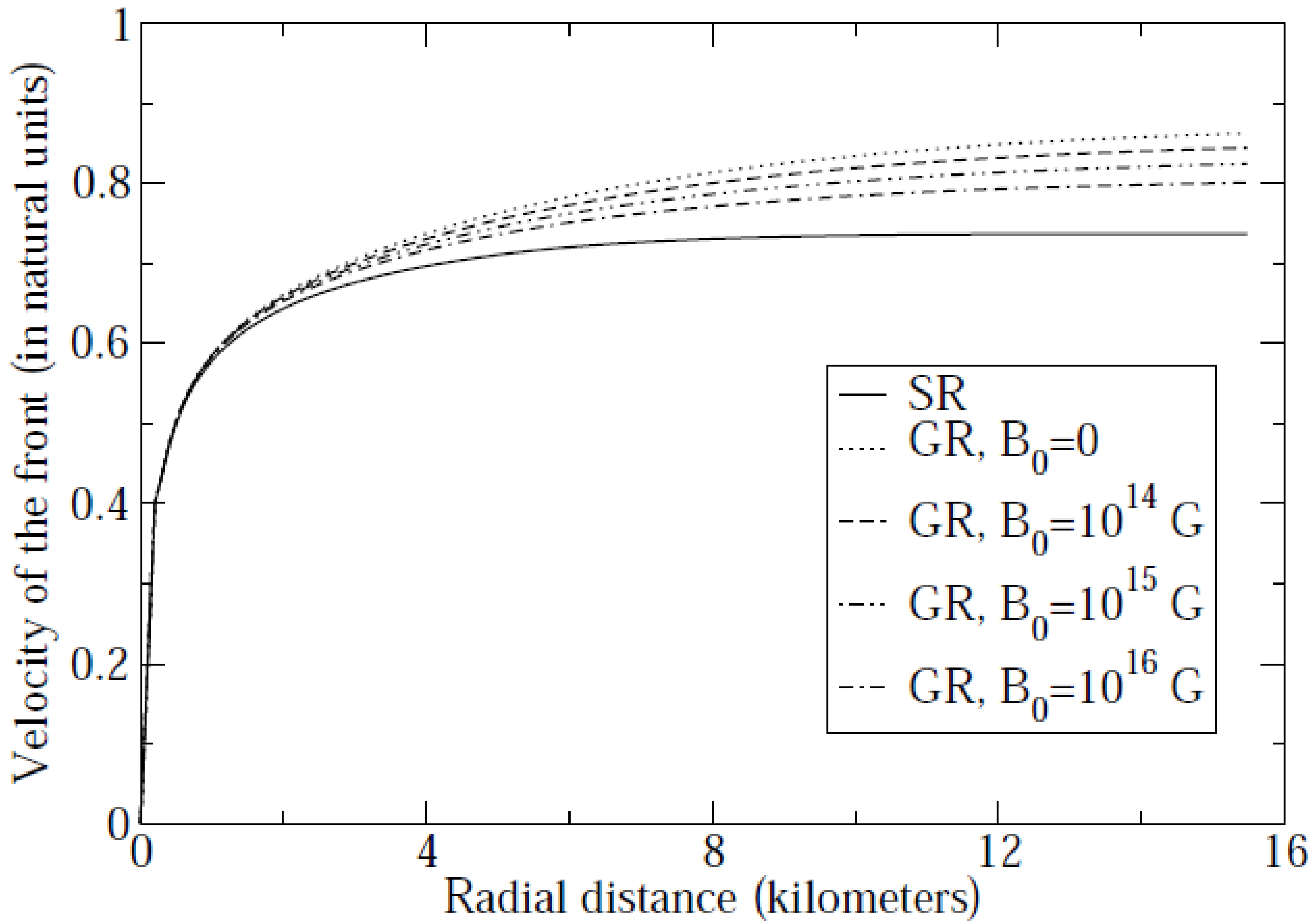
$$K = 2n(1+v^2G)\left(\frac{\partial \gamma}{\partial r} + \frac{\partial \alpha}{\partial r} + \frac{2}{r} + \frac{cot\theta}{r}\right)$$

$$K1 = -(1+G)\left(\frac{\partial \gamma}{\partial r} + \frac{\partial \rho}{\partial r}\right)$$

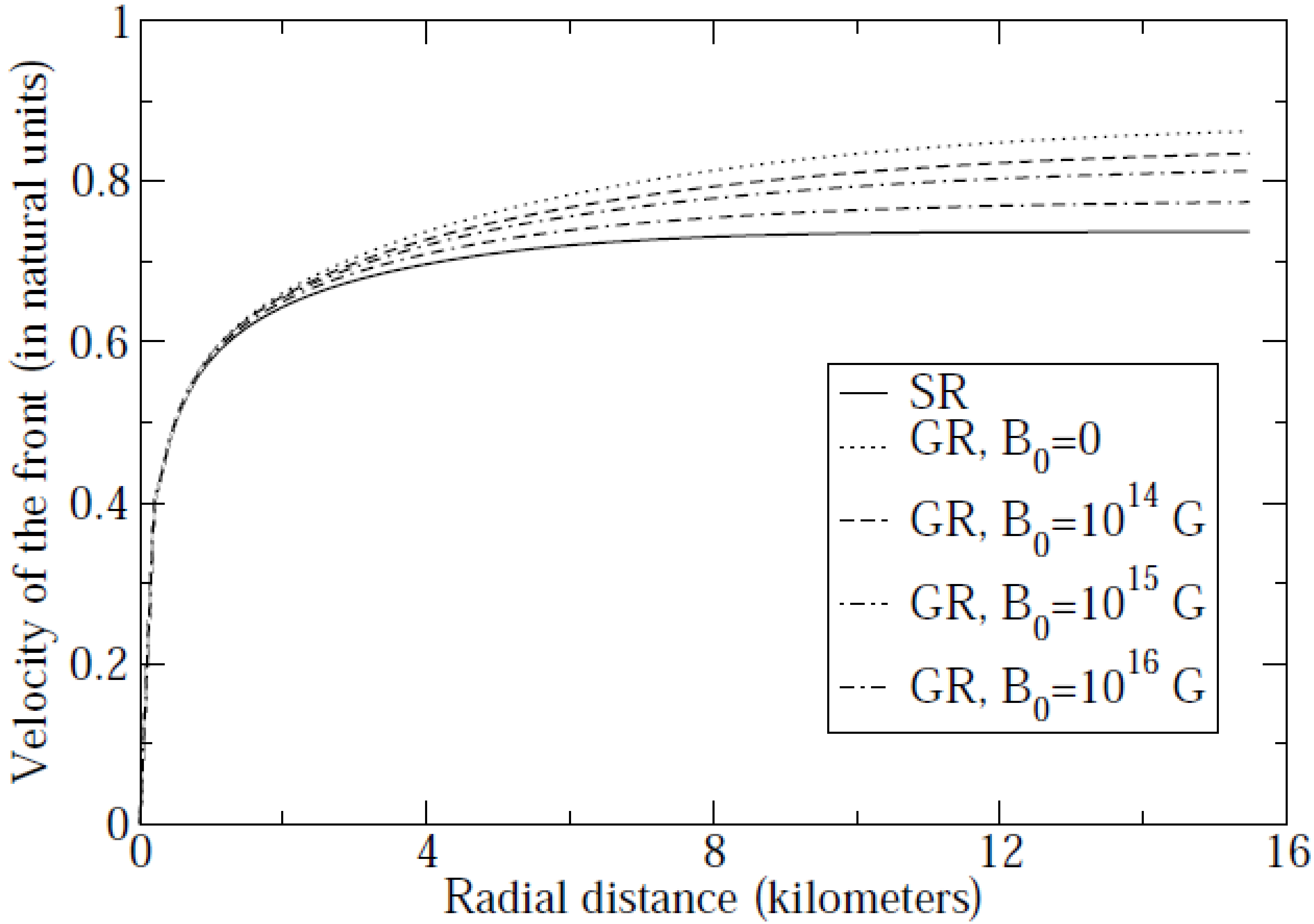
$$K2 = \frac{(1+G)(1+v^2G)}{4\pi} F_r$$

$$A = -1; \quad E = 0;$$

$$B = \frac{B1}{A1}; \quad C = \frac{vrsin\theta}{C1} \qquad A1 = e^{\gamma+\rho}; \quad B1 = -e^{\gamma+\rho}; \quad C1 = e^{\rho};$$



Variation of velocity of the conversion front along the radial direction for a non rotating star, with ζ = 0, for different surface field values.



Variation of velocity of the conversion front along the radial direction for a non rotating star, with ζ = 1, for different surface field values.

Rotating star

Canonical picture: we assume that the magnetic field of the NS is due to a dipole at the center of the star.

$$\vec{B} = \frac{\mu_0 m}{4\pi}\left(\frac{2cos\theta}{r^3}\hat{r} + \frac{sin\theta}{r^3}\hat{\theta}\right)$$
$$= B_0\left(\frac{2cos\theta}{r^3}\hat{r} + \frac{sin\theta}{r^3}\hat{\theta}\right)$$

$$\frac{dr}{d\tau} = vG \qquad G = \sqrt{\frac{e^{\gamma+\rho} - e^{\gamma-\rho}r^2\omega^2 sin^2\theta}{e^{2\alpha}}}$$

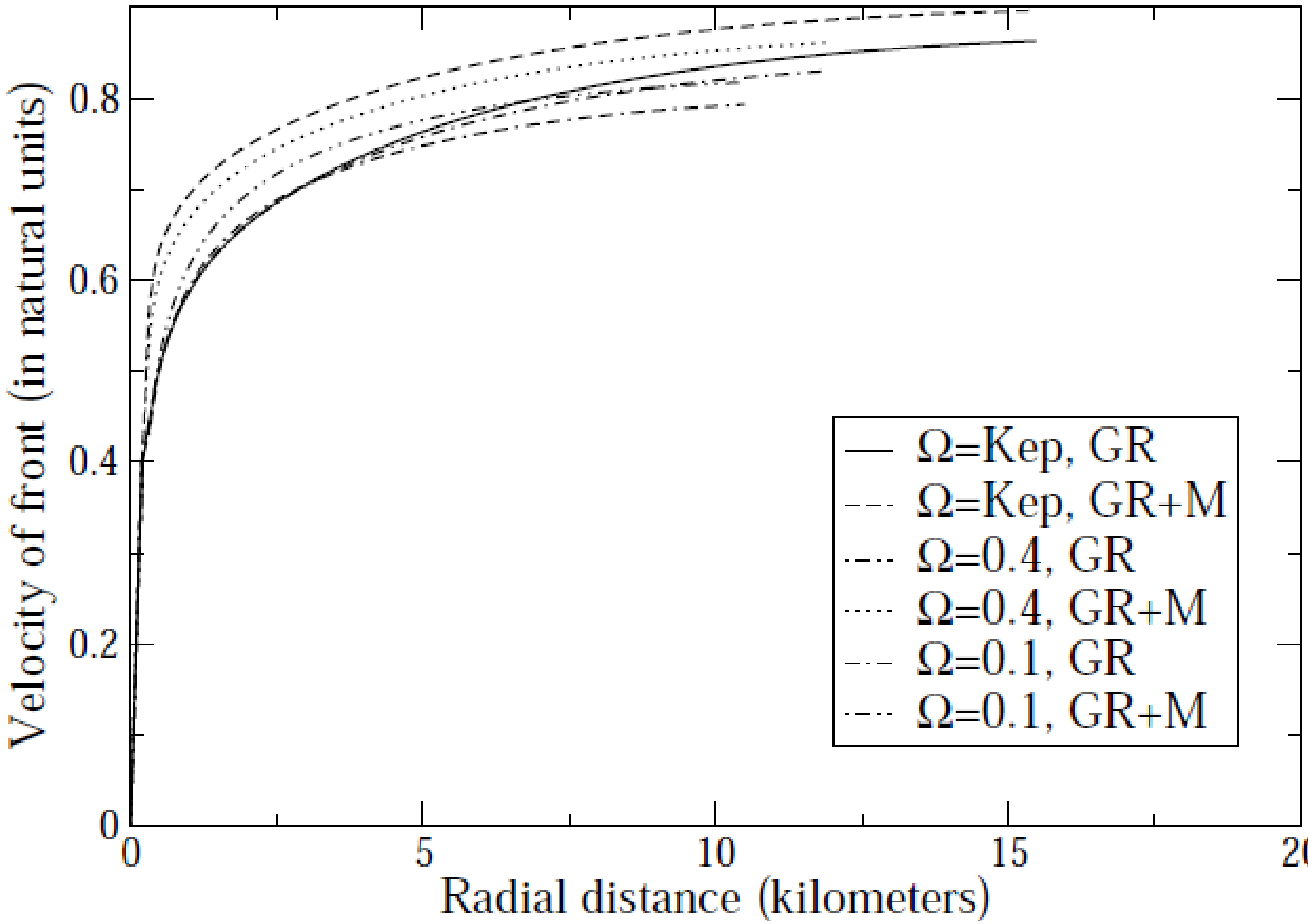
$$A = \frac{v\omega r sin\theta}{C1} - 1; \quad E = \frac{2\omega^2 r sin\theta}{C1} + \frac{2\omega^2 e^{\gamma-\rho} r sin\theta}{A1};$$

$$B = \frac{B1}{A1} - \frac{v\omega r sin\theta}{C1}; \quad C = \frac{2\omega e^{\gamma-\rho} r^2 sin\theta}{A1} + \frac{v r sin\theta}{C1}$$

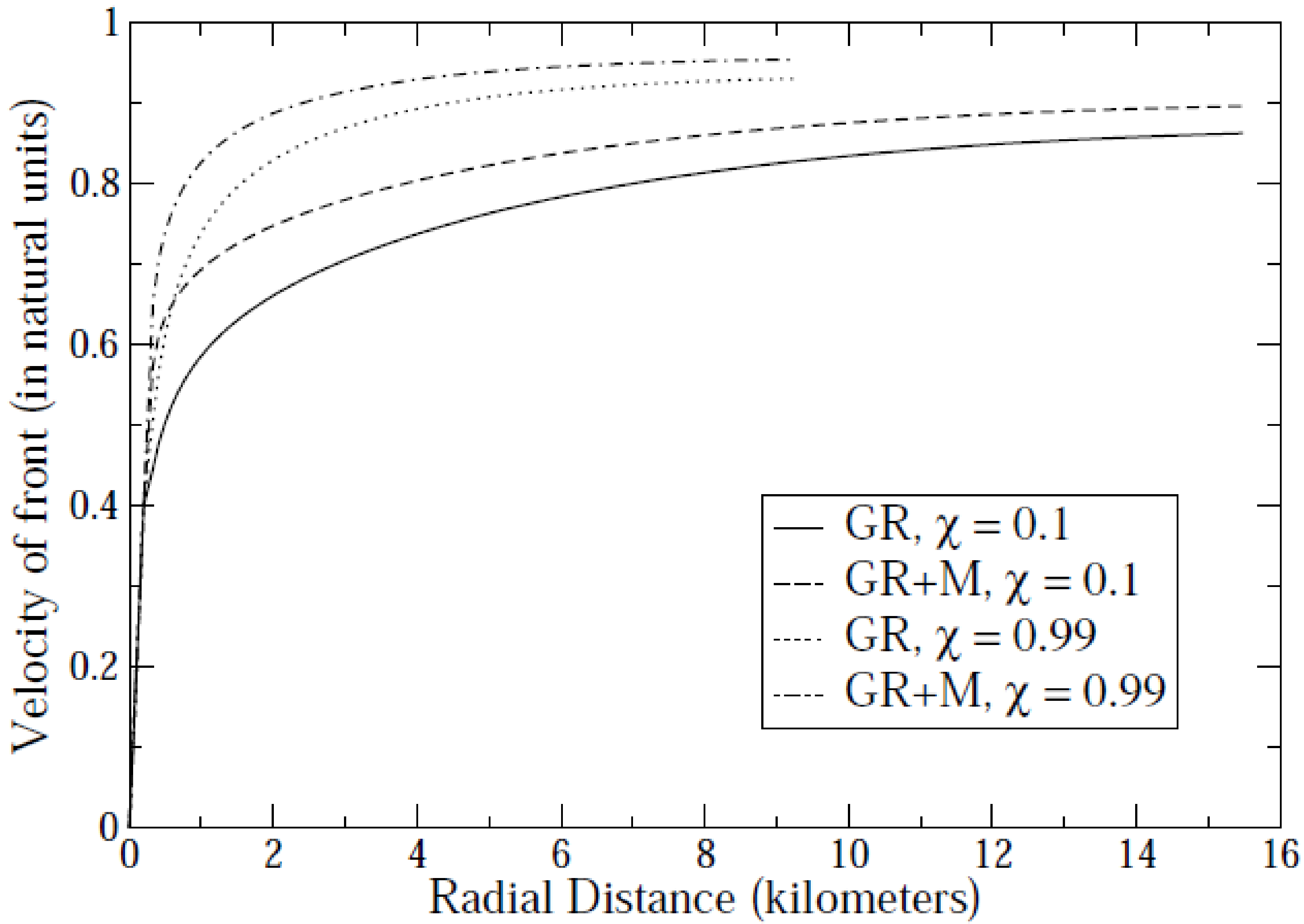
$$A1 = e^{\gamma+\rho} - e^{\gamma-\rho} r^2 \omega^2 sin^2\theta;$$

$$B1 = -e^{\gamma+\rho} - e^{\gamma-\rho} r^2 \omega^2 sin^2\theta;$$

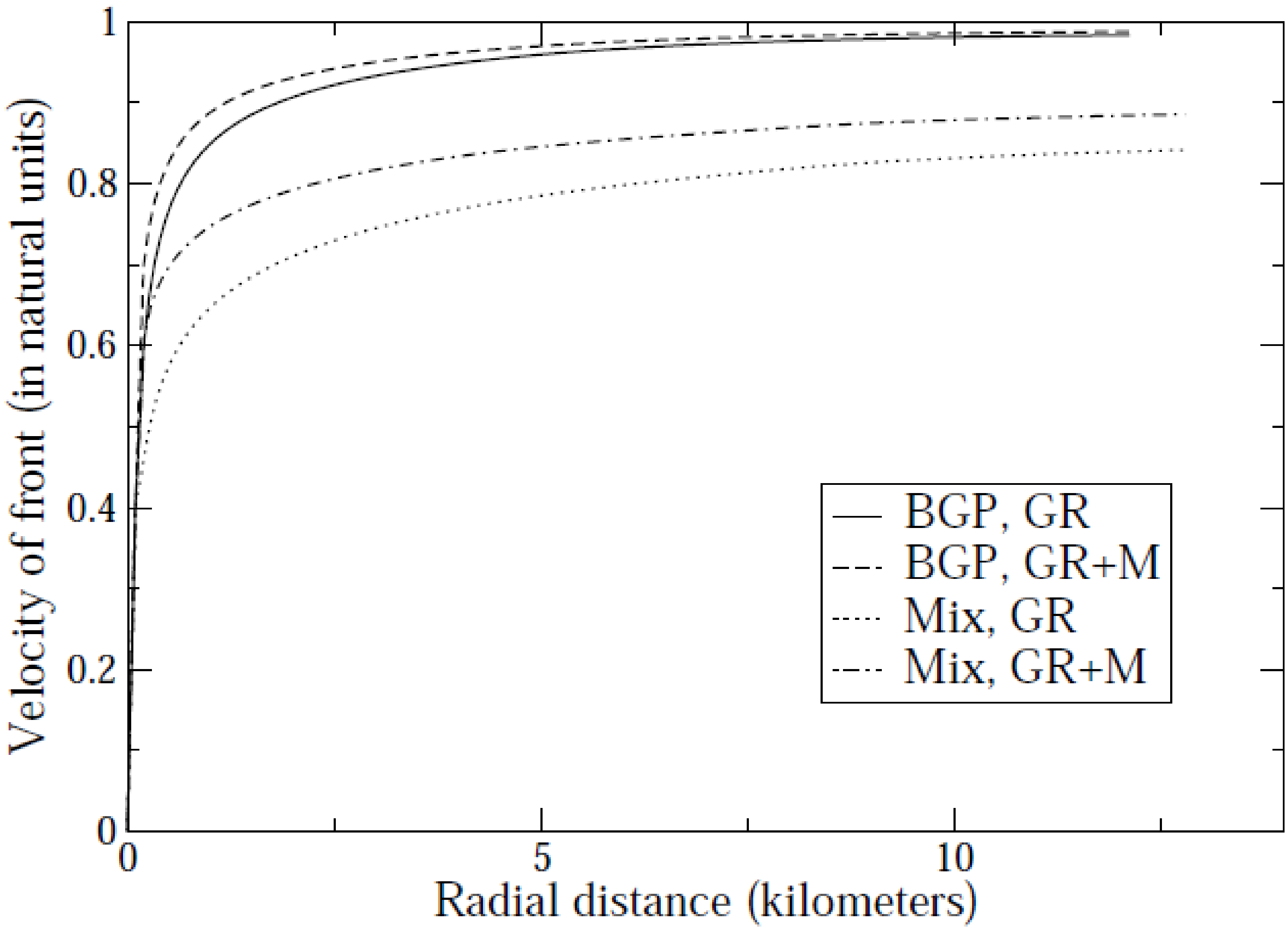
$$C1 = \sqrt{r^2 \omega^2 sin^2\theta - e^{2\rho}};$$



Variation of velocity of the conversion front along χ = 0.1 for the star rotating with three different velocities for two cases, GR and GR+M. Keplarion = $0.89\times10^4 sec^{-1}$ and other are given in the units of $10^4 sec^{-1}$.



Variation of velocity of the conversion front along two radial direction (χ = 0.1 and χ = 0.99) of the star rotating with Keplerian velocity.



Variation of the velocity of the conversion front along the equatorial direction (χ = 0.1) for the BGP and mixed EOS.

Alternate field configurations

Bocquet et al. 1995

$$B_r = B_0 \frac{2cos\theta}{e^{(\gamma-\rho)/2}}$$

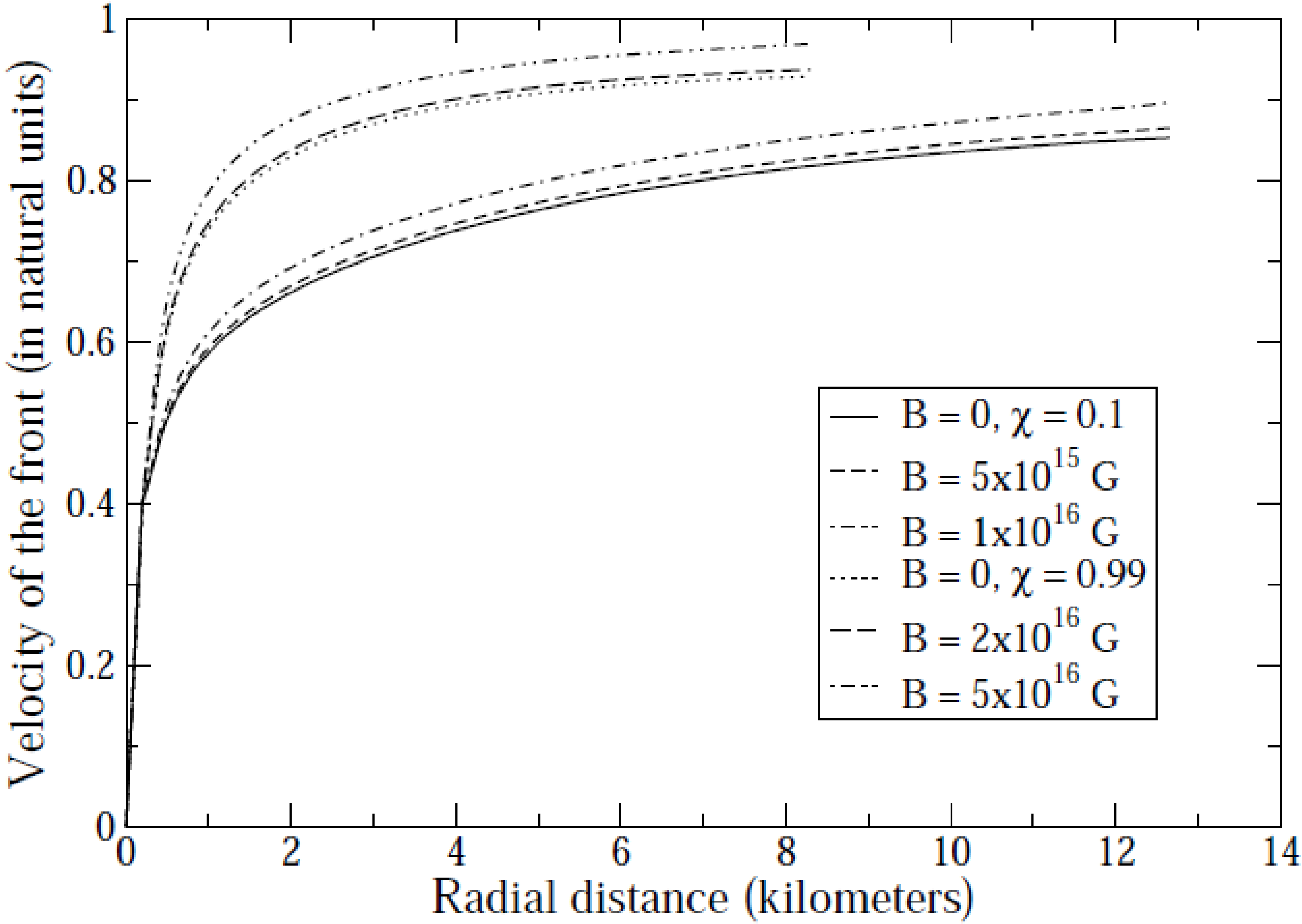
$$B_\theta = B_0 \frac{2sin\theta}{e^{(\gamma-\rho)/2}},$$

$$F_r = \frac{2{B_0}^2 sin\theta}{\mu_0 e^{(\gamma-\rho)}} \left[sin\theta \left(\frac{\partial\gamma}{\partial r} - \frac{\partial\rho}{\partial r} - \frac{2}{r} \right) + \frac{cos\theta}{r} \left(\frac{\partial\rho}{\partial\theta} - \frac{\partial\gamma}{\partial\theta} \right) \right]$$

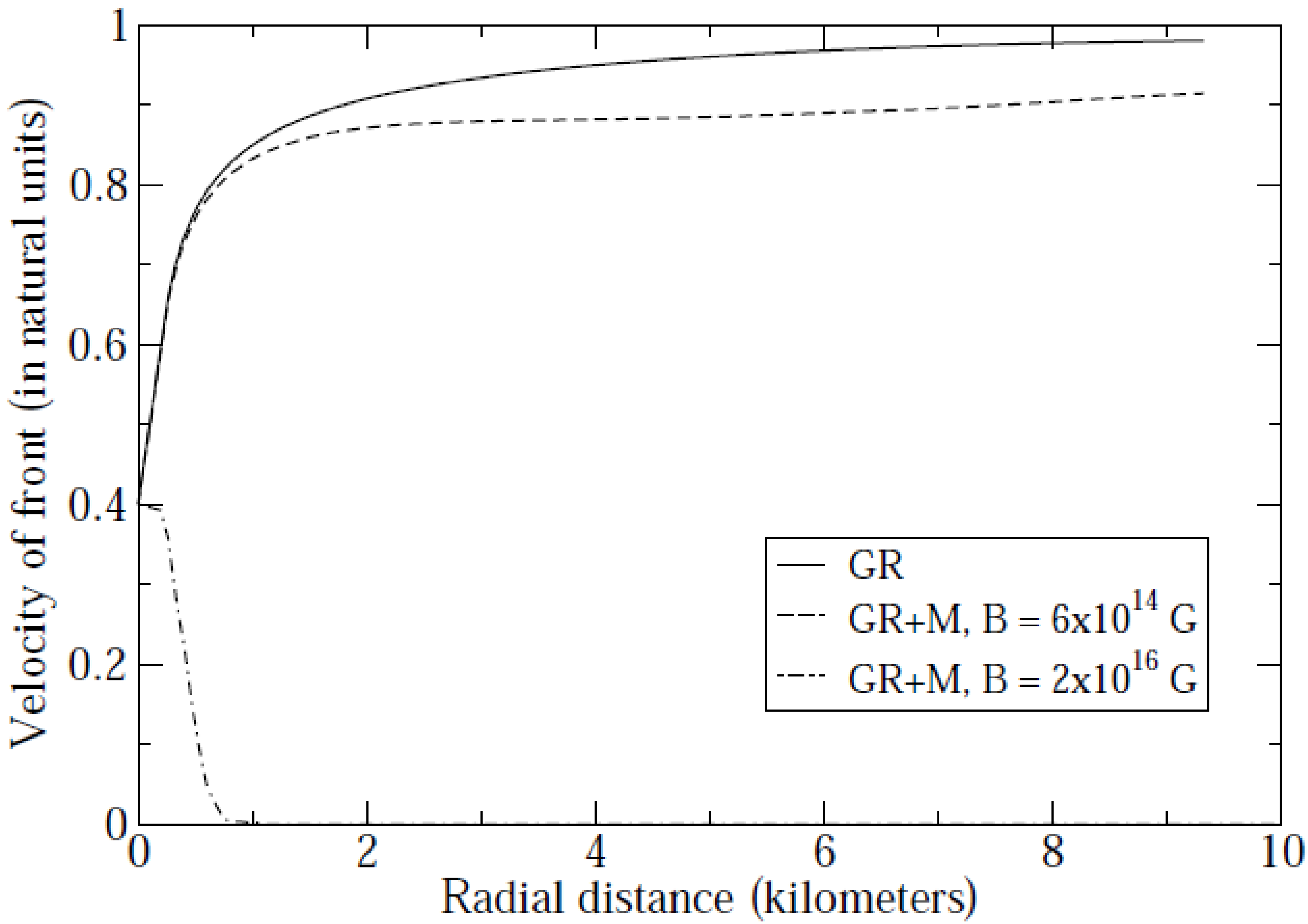
Field configuration proposed for magnetars

$$B_b(n_b/n_0) = B_s + B[1 - exp(-\beta(n_b/n_0)^\gamma)]$$

Chakraborty et al. 1997



Variation of the velocity of the conversion front with a different magnetic field configuration obtained from (Bocquet et al 1995; Konno et al. 1999)



Variation of the velocity of the conversion front with a different magnetic field configuration given by (Chakrabarty et al. 1997).

Magnetic field

Results

The magnetic field serves as a negative term.
The front gets stalled at a distance of 5km from the centre.
Neutron star gets converted to a star with quark core but nuclear matter outside.

HYBRID STAR

Discussion

- Effect of magnetic field in a NS is very complicated.

- The effect does not only depend on the strength of the field but also on its configuration.

- Considering a field created by a poloidal field at the center of the star the velocity of the front gets boosted by the field and the NS gets converted to QS.

- A slowly varying field along the star keeps the conclusion more or less same except for the fact that a higher surface magnetic field is needed for the same effect which can only be present in magnetars.

- On the other hand, a strong radial field configuration gives the totally opposite picture. The front gets stalled after some distance thereby stopping the conversion process. Therefore we have a star with a quark core surrounded by nuclear matter, which is referred as a hybrid star in the literature.

Discussion

- If the neutron star matter is superconducting, one can conjure up a scenario where the magnetic field inside the neutron star could be negligibly small, a possibility we have not considered here.

- Our results presented here give a qualitative understanding of the effect of the magnetic field on the conversion of NS to QS, but for quantitative estimates numerical simulation of the relativistic MHD code is needed.

- As the exact nature of the field configuration is not known, we need to carry out similar calculation with other complicated forms of magnetic field (namely both poloidal and toroidal) with different values of magnetic field.

- Furthermore such high magnetic fields can even modify the Einstein equation for the metric as the matter is then no longer an ideal fluid. All such a calculation is on our immediate agenda.

THANK YOU