

# Interaction of Radiation with Matter

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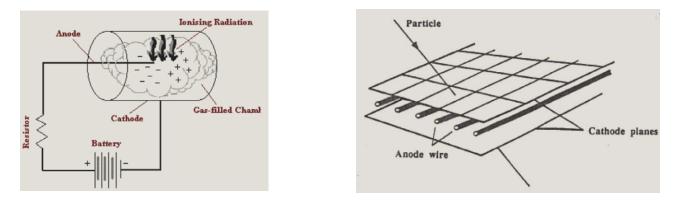
# **Radiation Detection**

Based on transfer of part or all of radiation energy to the detector mass where it is converted into some other form more accessible to human perception

### **Ionization detectors**

First electrical devices developed for radiation detection. Direct collection of the ionization electrons and ions produced in a gas by passing radiation, form an electric current signal

Proportional chamber – energy Multi-Wire Proportional Chamber (MWPC) – energy & position – *tracking* Drift Chamber – time – TPC (Drift+MWPC)



#### Scintillation detectors

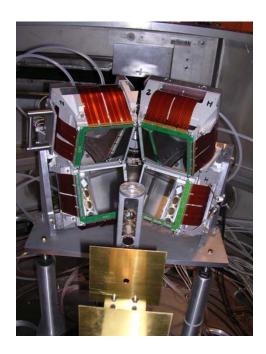
Certain materials when struck by ionizing radiation generates light in the scintillators. Coupled to an amplifying device (PMT) these scintillations can be converted into electrical pulses which can then be analyzed and counted electronically to give information concerning the incident radiation.

Inorganic material – NaI, BGO, PbWO<sub>4</sub> Organic material – Plastic

#### Semiconductor Detectors

Based on crystalline semiconductor materials (e.g silicon, germanium). Passage of ionizing radiation creates electron-hole pairs which are then collected by an electric field. Greater stopping power than gas detectors. Compact in size and can have very fast response times.



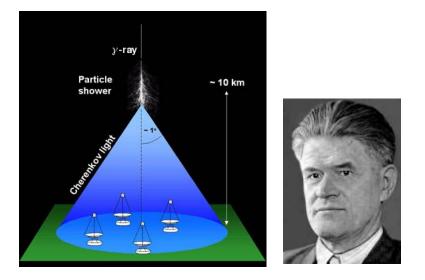


#### **Cerenkov Detectors**

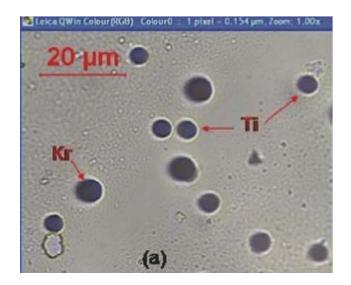
Particle detector for charged particles named after physicist Pavel Cherenkov. If the speed of charged particles in a medium exceeds the speed of light in this medium, they emit radiation (in optical light). The principle of a Cherenkov counter is based on the detection of this Cherenkov radiation.

### Solid State Nuclear Track Detectors

Ionization and excitation produced by the charged particles along their path through polymers cause molecular bonds to break, producing narrow damage trails. Damaged portions become chemically more reactive to suitable chemical reagents (etchants).



Pavel Alekseyevich Cherenkov (1904-1990)



### **Radiation Interactions**

Basic reactions which occur when radiation encounters matter and the effects produced by these processes

Operation of any **radiation detector** depends on the manner in which the radiation interacts with the detector material.

*Understanding this mechanism* : paramount importance. These processes are the basis of all particle detection devices and determines the *sensitivity* and *efficiency* of a detector.

Penetrating radiation sees matter in terms of aggregate of electrons and nuclei (and their constituents). Type of interaction will depend on the type of radiation, its energy and the type of material.

Radiations can be **charged particles** 

- (a) <u>heavy charged particles</u> :  $\mu$ ,  $\pi$ , p,  $\alpha$ -particles or heavier nuclei
- (b) electrons and positrons

Continuously interact through the Coulomb force with the electrons in the medium through which they pass.

Radiations can be **uncharged** n, x-rays and  $\gamma$ -rays

Not subjected to Coulomb force. Neutral particles first have to produce charged particles, which are consequently detected thru their interactions.

### **Interaction of heavy charged particles**

Primarily interact with matter through Coulomb forces with the orbital electrons within the absorber atoms. Entering the medium, the charged particle immediately interacts simultaneously with many electrons. In any one such encounter, the electron feels an impulse from the Coulomb force as the particle passes its vicinity.

Depending on the proximity of the encounter, this impulse may be sufficient *either to raise the electron to a higher shell*: **excitation**/ soft collision *or to remove it completely from the atom* : **ionization**/ hard collision. In some hard collisions, enough energy is transferred to the electrons such that the electron itself causes substantial secondary ionization.

Interactions of the particles with nuclei occur rarely (not significant in the detector response). Typically  $\sigma_{inel} \sim 10^{-16} \text{ cm}^2 >> \sigma_{nucl scatt} \sim 10^{-24} \text{ cm}^2$  (barn) *dominant energy loss due to atomic electron collisions* 

The maximum energy that can be transferred to the electron of mass m from a charged particle of mass M with kinetic energy E in a single collision is 4E(m/M), or about 1/500 of its energy for a proton. At any given time, the particle is interacting with many electrons, so the net effect is to *decrease its velocity continuously until the particle is stopped*.

Amount of energy lost in a single collision  $\delta E \ll E$ , particle undergoes large no. of inelastic collisions ( $N \gg 1$ ), process statistical in nature, occurring with a certain quantum mechanical probability.

Because their number per macroscopic path length is generally large, the fluctuations in the total energy loss are small and one can meaningfully work with the average energy loss per unit path length : *stopping power* dE/dx and the *range* of the penetrating particles (distance beyond which no particle will penetrate).

*e.g* 10 MeV proton already loses all of its energy in 0.25 mm of Copper

# **Stopping Power**

The quantum mechanical calculation of stopping power *S Bethe-Bloch formula* : basic expression used for energy loss calculations.

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{mv^2} NZ \left[ \ln \frac{2mv^2}{I} - \ln \left( 1 - \frac{v^2}{c^2} \right) - \frac{v^2}{c^2} \right]$$

v - velocity, *ze*- charge of the incident particle, *N*, *Z* - number density, atomic number of the absorber atoms, *m* - electron rest mass, *e*- electronic charge, *I*- average excitation and ionization potential of the absorber (experimentally determined parameter for each element)

Nonrelativistic particle ( $v \ll c$ ): only first term in [] significant. It varies slowly with particle energy, thus general behavior is given by the multiplicative factor.

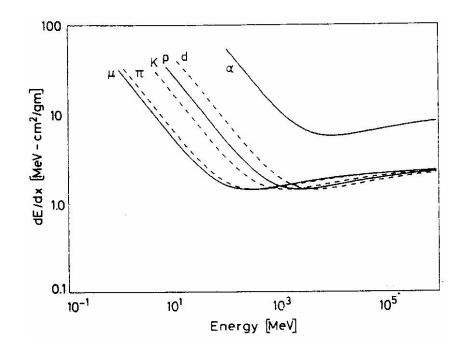
For a given nonrelativistic particle, S varies as  $1/v^2$ . A particle with lower velocity spends more time in the vicinity of any electron, imparting more impulse to it  $\rightarrow$  incoming particle loses more energy.

S varies as  $z^2$  of the incoming particle. So for a given velocity the **particle with more charge will lose more energy** in a given material. Used in *identifying charged particles* in expts. ( $\alpha$ -particle will lose energy at a faster rate than proton of same velocity).

$$-\frac{dE}{dx} \propto \frac{z^2}{\beta^2} \sim M \frac{z^2}{E}$$

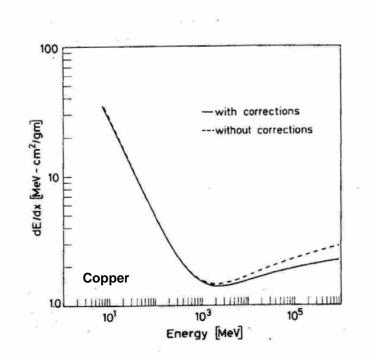
The material with higher NZ will be a better absorber (higher stopping power)  $\rightarrow$  lead is used as a very effective absorber.

Limitations : valid as long as velocity of incoming particle remains large compared with orbital electron velocities in the absorbing material. At very low velocity (E/A < 1 MeV) the positively charged particle tends to **pick up electrons** which effectively reduces its charge ( $Z_{eff} < Z$ ), and finally it becomes a neutral atom. Reducing effective charge reduces the electronic energy loss. At these energies nuclear stopping power dominates.



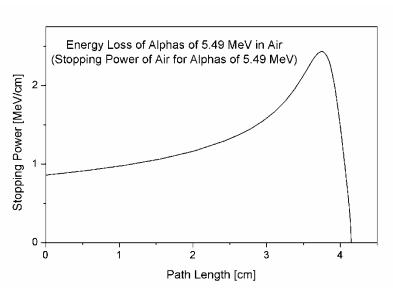
Bethe-Bloch formula as function of kinetic energy for different particles

- Non-relativistic energies : dE/dx dominated by the over all  $1/\beta^2$  factor and decreases with increasing velocity. At  $v \sim 0.96c$  (particles minimum ionizing).  $(dE/dx)_{min}$  almost same for particles of same charge. Beyond this point, the term  $1/\beta^2$  almost constant and dE/dx rises again due to the log term.
- For energies below minimum ionizing value, each particle exhibits a distinct dE/dx curve particle identification
- Very low energy region (not shown) : *Bethe-Bloch formula* breaks down. *dE/dx* reaches a maximum and drops sharply (particle picks up electrons)



**Density correction** : Electric field of the particle tends to polarize atoms along its path, **shielding electrons** (far from the particle path) from full electric field intensity. Collisions with these outer lying electrons contribute less to energy loss. Important at high energy (effect depends on material *density*).

**Shell correction** : velocity of incident particle comparable to orbital velocity of bound electrons ( $v_{inc} \sim v_{orbit}$ ). At low energy, correction to the stopping power arises since the assumption of stationary target electron is not valid and stopping power is reduced. Depends on electron binding energy.



**Bragg Curve** 

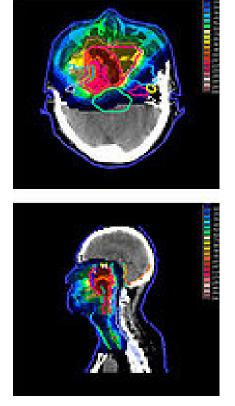
For most of track, dE/dx increases roughly as 1/E. Near the end of track – charge reduced through electron pickup and the curve falls off.

Variation of dE/dx as a function of penetration depth of the particle in matter

#### Particle loses most of its energy near the end of its path

This fact is utilized in **medical applications** of radiation where high radiation dose delivered to deeply embedded malignant growths with minimum destruction to overlaying tissue.

# **Proton therapy**



Irradiation of nasopharyngeal carcinoma

Proton therapy uses a beam of protons to irradiate diseased tissue, most often in the treatment of cancer. The dose delivered to tissue is maximum just over the last few millimeters of the particle's range : **Bragg peak** 

#### Advantage

Ability to more precisely localize the radiation dosage when compared with other types of external beam radiotherapy.

Stays focused on the tumor shape and delivers only low-dose side-effects to surrounding tissue.

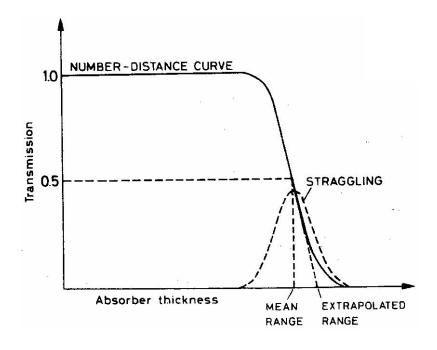
Skin exposure at the entrance point is higher, but tissues on the opposite side of the body than the tumor receive no radiation.

Protons of different energies with Bragg peaks at different depths are applied to treat the entire tumor- **blue lines**. The total radiation dosage of the protons is called the Spread-Out Bragg Peak (SOBP) - **red line**.

# **Range of charged particles**

Depends on material type, particle type and its energy. Range of charged particles of a given energy is thus a fairly **unique quantity** in a specific absorber material.

Pass a beam of particles at the desired energy through different thicknesses of a material and measure the ratio of transmitted to incident particles.



Small thickness : nearly all particles pass through. Near the range, ratio slopes down over a certain spread of thicknesses. *Two identical particles with the same initial energy will not suffer same number of collisions and hence same energy loss*. A measurement with an ensemble of identical particles will show a statistical distribution of ranges centered about some mean value : range straggling

The details of microscopic interactions undergone by any specific particle passing through a thickness of matter, vary somewhat randomly, its energy loss is a statistical process. Fluctuations occur in the number of collisions suffered and in the energy transferred in each collision. Therefore a spread in energies always results after a beam of monoenergetic charged particles has passed through a given thickness of absorber. The width of this energy distribution is a measure of energy straggling, which varies with the distance along the particle track.

Same problem viewed from different angles :

Fluctuations in thickness of path length for a fixed loss in energy. Fluctuations in energy loss for a fixed thickness of absorber.

### **Interaction of electrons**

The basic mechanism of collision loss is also valid for electrons/positrons, the Bethe-Bloch formula must be modified due to

*Small mass* – Large deviations in the path possible because it is interacting with another (orbital) electron and a much larger fraction of its energy can be lost in a single encounter. Sometimes electron-nuclear interactions occur, abruptly changing the electron direction.

For electrons the collisions are between *identical particles* (indistinguishability)

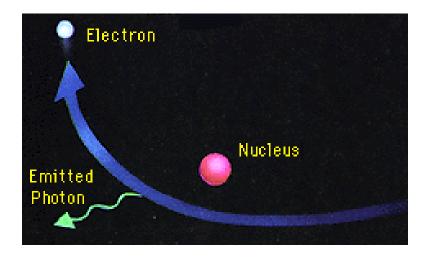
Collisional losses for electrons

$$-\left(\frac{dE}{dX}\right)_{c} = \frac{2\pi e^{4}NZ}{mv^{2}} \left[\ln\frac{mv^{2}E}{2I^{2}\left[1-\beta^{2}\right]} - \left(\ln 2\right)\left[2\sqrt{1-\beta^{2}}-1+\beta^{2}\right] + \left(1-\beta^{2}\right) + \frac{1}{8}\left(1-\sqrt{1-\beta^{2}}\right)^{2}\right]$$

Concept of range is less definite for fast electrons than for HCP : *electron total path length* is considerably greater than the distance of penetration along the initial velocity vector. Electron range is taken from a transmission curve plot by extrapolation of the linear portion to zero (absorber thickness required to assure that almost no electrons can penetrate the entire thickness). Fast electrons lose energy at a lower rate compared with HCP. For equivalent energy, the specific energy loss of electrons is much lower than HCP. *Electron ranges : 1-2 mm per MeV*.

The Coulomb forces that constitute the major mechanism of energy loss for both electrons and HCP are present for positive or negative charge on the particle. Whether the interaction involves a repulsive or attractive force between the incident particle and orbital electron, the *impulse and energy transfer for particles of equal mass are about the same*.

For positrons the *annihilation radiation* is generated at the end of the track. Because these 0.511 MeV photons are very penetrating compared with the range of the positron, they can lead to the deposition of energy far from the original positron track. For electrons/positrons energy loss through electromagnetic radiation arising from scattering in the electric field of a nucleus (**bremsstrahlung**) also becomes important. These can emanate from any position along the track.



Any charge must radiate energy when accelerated and the deflections of the electron in its interactions with the absorber correspond to such acceleration.

The specific energy loss due to bremsstrahlung is

$$-\left(\frac{dE}{dx}\right)_{r} = \frac{NEZ(Z+1)e^{4}}{137m^{2}c^{4}} \left(4\ln\frac{2E}{mc^{2}} - \frac{4}{3}\right)$$

The total stopping power :

$$\frac{dE}{dx} = \left(\frac{dE}{dx}\right)_c + \left(\frac{dE}{dx}\right)_r$$

$$\begin{array}{cc} \textit{collisional} & \textit{radiative} \\ \infty & Z & \infty & Z^2 \end{array}$$

The ratio of the specific energy losses is :

$$\frac{\left(\frac{dE}{dx}\right)_{r}}{\left(\frac{dE}{dx}\right)_{c}} \cong \frac{EZ}{700}$$

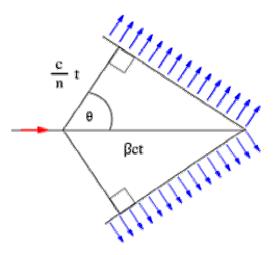
where E is in MeV. Clearly for electrons with very *high energy*, radiative processes are more significant than ionization and excitation specially in absorber materials of *high atomic number*.

### **Bremsstrahlung**

- $E \sim \text{MeV}$  : a relatively small factor
- $E \sim 10 \text{ MeV}$  : radiation loss  $\sim \text{ collision loss}$
- $E \sim 100$  GeV, electrons and positrons only particles in which radiation contributes substantially to the energy loss. The emission probability  $\sim 1/m^2$ . Radiation loss by muons (m = 106 MeV) is thus some 40,000 times smaller than electrons.

### **Cerenkov radiation**

Cerenkov radiation arises when a charged particle in a material medium moves faster than the speed of light in that same medium ( $\beta c = v_{\text{particle}} > c/n$ , n – index of refraction, c – speed of light in vacuum). An electromagnetic **shock wave** similar to a sonic shock wave is generated. The coherent wavefront formed is conical in shape and is emitted at a well-defined angle  $\cos\theta = 1/\beta n$ w.r.t the trajectory of the particle.



The energy loss increases with  $\beta$ , however even at relativistic energies the energy loss is small compared to collision loss.

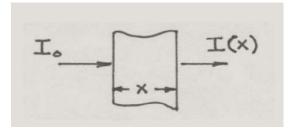
<u>Advantage</u>: very accurate  $\beta$  measurement of relativistic particles, since cone angle of radiation ( $\theta$ ) depends on  $\beta$ . The radiation is observed only for  $\beta$  above threshold. The **Cerenkov detectors** based on this principle are widely used in high energy physics, study of cosmic rays, neutrinos etc.

# **Interaction of** γ**-rays**

Photon's lack of an electric charge makes impossible the many inelastic collisions with atomic electrons so characteristic of charged particles. They are many times more **penetrating** in matter than charged particles. A beam of photons is not degraded in energy as it passes through a thickness of matter, only **attenuated in intensity**. They interact with matter in mainly three ways, one dominant over other depending on the  $\gamma$ -ray energy.

#### Photoelectric absorption, Compton scattering, Pair production

All these processes lead to the partial or complete transfer of the  $\gamma$ -ray photon energy to electron energy. Each of the interaction processes removes the  $\gamma$ -ray photon from the incident direction either by absorption or by scattering away. The probability that the  $\gamma$ -ray photon is removed from the beam is called linear attenuation coefficient  $\mu$  and is given by  $I(x) = I_0 e^{-\mu x}$  $\mu = N\sigma$ , N: no. density of atoms,  $\sigma$ : total interaction cross section



Attenuation of a beam of gamma radiation through an absorber of thickness x

very different from charged particle slowing down gradually **Photoelectric absorption** : the absorption of a photon by an atomic electron with the subsequent ejection of the electron from the atom. Outgoing electron energy E = hv - B.E. B.E is binding energy of the electron. Since a free electron cannot absorb a photon and also conserve momentum, *photoelectric effect always occurs on bound electrons with the nucleus absorbing the recoil momentum* 

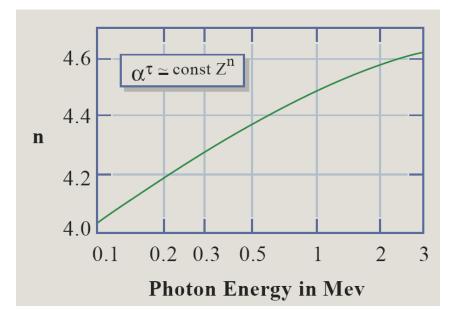
Predominant mode of  $\gamma$ -interaction in all kinds of matter, especially the high-Z absorbers, at energies less than ~ 0.1 MeV.

(Typical ionization potentials of K electrons are 2.3 keV (Al), 10 keV (Cu), and ~ 100 keV (Pb)).

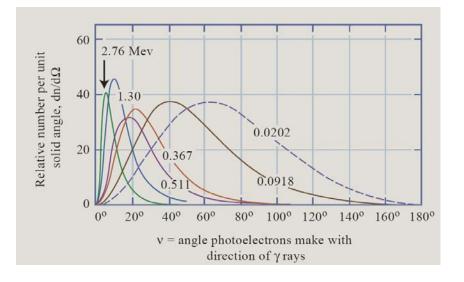
The cross-section of Photoelectric absorption is given by

$$\frac{d\sigma}{d\Omega} = 4\sqrt{2} \frac{r_e^2 Z^5}{(137)^4} \left[\frac{m_e c^2}{\hbar\omega}\right]^{7/2} \frac{\sin^2\theta\cos^2\varphi}{\left[1 - \frac{v}{c}\cos\theta\right]^4}$$

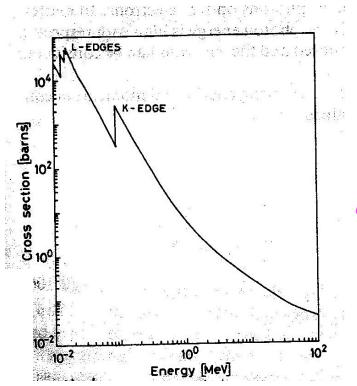
v is the photoelectron speed,  $\theta$ ,  $\phi$  are angles specifying the direction of photoelectron



#### Variation with energy of incident photon of the exponent n in the total cross section for photoelectric effect.



Angular distribution of photoelectrons for various incident photon energies. The peak moves toward forward direction as the energy increases, a behavior which can be qualitatively obtained from the  $\theta$ -dependence





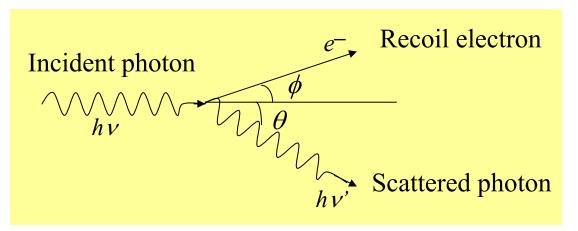
At energies above the highest electron BE of the atom (K shell), the cross section is relatively small but increases rapidly as the K-shell energy is approached. Just after this point, the cross section drops drastically (discontinuous jump) since the K-electrons are no longer available for the photoelectric effect (K absorption edge). Below this energy, the cross section rises once again and dips as the L, M levels etc are passed (L-absorption edges, M-absorption edge). The effect is more pronounced in the high-Z material.

### **Compton scattering** :

interaction process of a  $\gamma$ -ray photon with a free electron in the absorbing material. If the photon energy is much higher than the binding energy of a bound electron, it can be considered to be a free electron. The incoming  $\gamma$ -ray photon is deflected through an angle  $\theta$  w.r.t. to its original direction.

### The photon transfers a portion of its energy to the (recoil) electron.

Because all angles are possible, the energy transfer vary from zero to a large fraction of the  $\gamma$ -ray energy.



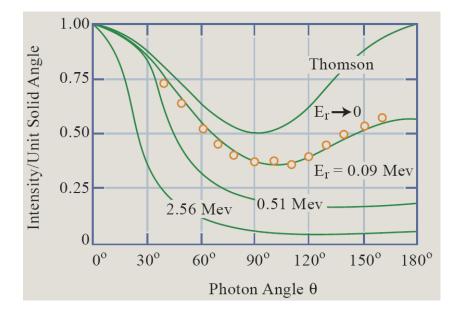
 $hv' = hv/[1+\alpha(1-\cos\theta)]$   $cot\phi = (1+\alpha) \tan(\theta/2)$ where  $\alpha = hv/mc^2$ 

 $\theta \cong 0$ , very little energy transferred,  $\theta \cong \pi$ , maximum energy transferred ( $\gamma$ -ray backscattered) The sharp drop at the maximum recoil energy : *Compton edge* 

The cross-section of Compton scattering is given by *Klein-Nishina formula*:

$$\frac{d\sigma}{d\Omega} = Zr_o^2 \left[\frac{1}{1+\alpha\left(1-\cos\theta\right)}\right]^2 \left[\frac{1+\cos^2\theta}{2}\right] \left[1+\frac{\alpha^2\left(1-\cos\theta\right)^2}{\left(1+\cos^2\theta\right)\left[1+\alpha\left(1-\cos\theta\right)\right]}\right]$$

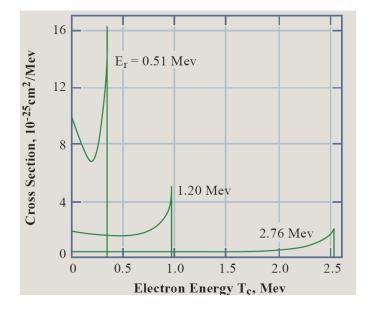
where  $\alpha \equiv hv / mc^2$ , measure of the photon energy in units of the electron rest mass energy and  $r_0 = e^2 / mc^2$  is the classical electron radius



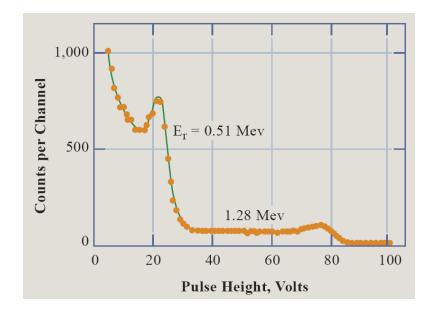
Thomson scattering - photon energy much lower than rest mass energy, scattering by a free electron then becomes elastic (no energy loss).

Angular distribution of Compton scattering at various incident energies  $E_r$ . All curves are normalized at  $0^\circ$ .

At any given  $\alpha$  the angular distribution is peaked in the forward direction. As  $\alpha$  increases, the forward peaking becomes more pronounced. The deviation from Thomson scattering is largest at large scattering angles. In practice the Klein-Nishina cross section has been found to be in excellent agreement with experiments at least out to  $hv = 10 mc^2$ .



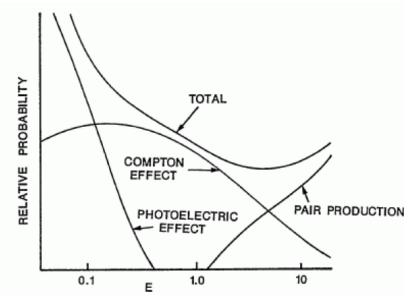
Energy distribution of Compton electrons for several incident gamma-ray energies.



Pulse-height spectra of Compton electrons produced by 0.51- and 1.28-MeV gamma rays.

Relative magnitudes of the distributions match quite well between calculation and experiment. The **distribution peaks near the cutoff** because there is an appreciable range of  $\theta$  near  $\theta = \pi$  (cosine changes slowly in this region) and so *hv* remains close to  $mc^2/2$ . This feature is **reminiscent of the Bragg curve** depicting the specific ionization of a charged particle. Pair production : Transformation of a photon into an *electron-positron pair*. To conserve momentum, this can only occur in the presence of a third body, usually a nucleus. To create the pair the photon must have at least an energy of 1.022 MeV. Any excess energy carried by the photon goes into the kinetic energy shared by the pair.

This positron subsequently annihilates after slowing down in the absorbing medium. For typical energies, both the  $e^-$  and  $e^+$  travel a few mm before losing all the kinetic energy. At this point the  $e^+$  combines with an  $e^-$  and *two annihilation photons* (0.511 MeV each) are produced, virtually in coincidence with the pair production interaction. The probability of pair production per nucleus  $\infty Z^2$  of the absorber material and rises sharply with energy.



*Relative probability of three processes* 

Total  $\sigma = \sigma_{\text{PE}} (\infty Z^{4.5}) + \sigma_{\text{C}} (\infty Z) + \sigma_{\text{pair}} (\infty Z^2)$ 

### **Interaction of Neutrons**

Neutrons do not carry electric charge, so they can not interact in matter by means of Coulomb force. Principal means of interaction is through the strong force with nuclei of the absorbing material (for which it must come within  $\sim 10^{-13}$  cm of the nucleus). Since matter is mostly empty space, the neutron is observed to be a *very penetrating particle* traveling many centimeters of matter without any interaction.

*Elastic scattering* A(n,n)A. Principal mechanism of energy loss for  $E_n \sim \text{MeV}$ *Inelastic scattering*  $A(n,n')A^*$ 

*Neutron capture* (n,p), (n,t), (n,d) etc at eV to keV region, charged particles emitted, cross section falls as 1/v

*Fission* most likely at thermal energies  $\sim 0.01 \text{ eV}$ 

*Radiative neutron capture*  $n + (A, Z) \rightarrow \gamma + (A+1, Z)$ , probability varies as 1/v*High energy hadron shower production* at E > 100 MeV Secondary radiations resulting from neutron interactions are mainly heavy charged particles produced by *neutron-induced nuclear reactions* or *nuclei of absorbing material* gaining energy from neutron collisions. Most neutron detectors utilize some type of conversion of incident neutron into secondary charged particles, which can then be detected directly.

Slowing down of neutrons (moderation) is most efficient when it interacts with protons or light nuclei. Scattered neutron

Incoming neutron

 $\theta$ Recoil nucleus ( $E_{\rm R}$ )  $E_R = E_n \frac{4A}{\left(1+A\right)^2} \cos^2 \theta$ 

head-on collision ( $\theta = 0^{\circ}$ ) :  $E_R \Big|_{\max} = E_n \frac{4A}{(1+A)^2}$ 

where A = mass of the target nucleus

 $E_{\rm n}$  = kinetic energy of the incoming neutron

 $E_{\rm R}$  = kinetic energy of the recoil nucleus

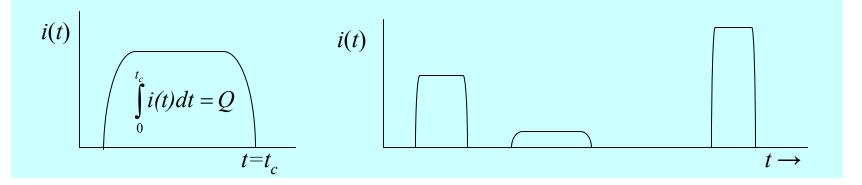
 $\theta$  = scattering angle of the recoil nuclei

Clearly only with proton & H-nuclei with A=1, a neutron can transfer all its energy in a single collision.

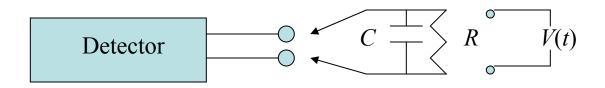
### **General Characteristics of Radiation Detector**

Many types of radiation detectors, same fundamental principle : *transfer of part or all of the energy to the detector mass where it is converted into some form accessible to human* and the transfer happens through one of the mechanisms discussed earlier.

Interaction time is very small (~ ps in solids to ~ ns in gases). Net result is the *appearance of a given amount of electric charge* within the detector volume, collected to form the basic **electrical signal**. Collection of charge is accomplished by imposing an electric field within the detector, which causes the charges to flow. The time required to fully collect the charge varies from one detector type to another, from *ms* to *ns*. We assume that the rate is low enough that the current due to many events are distinguishable.



The circuit normally used is of the following form:



Time required for signal pulse to reach its maximum value depends on  $t_c$  the intrinsic charge collection time of the detector. It can not be changed by any external circuit. But the decay time of the pulse is determined by the time constant *t* of the circuit.

Also the amplitude of the pulse  $V_{\text{max}} = Q / C$ . Thus since the capacitance is normally fixed, the amplitude of the signal pulse is directly proportional to the corresponding charge generated within the detector.

If Q is proportional to the energy of the incident particle, then the *pulse amplitude is a measure of the energy of the incident particle*.

## **Summary**

Radiation interaction with matter in which the basic mechanisms of charged particle, neutron and gamma interactions were discussed.Regardless of the type of nuclear radiation, the interactions taking place in a material medium invariably result in ionization and excitation which then can be detected.

- *Heavy charged particles and electrons* produce ion pairs in ionization chambers, or light emission (excitation of atoms) in scintillation counters, or electron-hole pairs in semiconductor detectors.
- *Neutrons* collide with protons which recoil and produce ionization or excitation.
- In the case of *gammas*, all 3 processes we have just discussed give rise to energetic electrons which in turn cause ionization or excitation.

Thus the basic mechanisms of nuclear radiation detection involve measuring the ionization or excitation occurring in the detector in a way to allow one to deduce the energy of the incoming radiation.

### References

### *Radiation detection and Measurement* G.F. Knoll (Addison-Wesley)

*Techniques in Nuclear Physics Experiment* W.R. Leo (Springer)